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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1248

INVESTIGATIONS OF LATERAL STABILITY OF A GLIDE BOMB USING AUTOMATIC CONTROL HAVING NO TIME LAG

By E. W. Sponder

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INVESTIGATIONS OF LATERAL STABILITY OF A GLIDE BOMB USING AUTOMATIC CONTROL HAVING NO TIME LAG*

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Outline:

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*"Untersuchung der Seitenstabilität einer Gleitbombe mit einer automatischen Steuerung ohne Voreilung." Zentrale für wissenschaftliches Berichtswesen der Luftfahrtforschung des Generalluftzeugmeisters (ZWB) Berlin-Adlershof, Forschungsbericht Nr. 1819, May 1943.

I. SYMBOLS

In addition to the symbols of flight mechanics according to DIN L 100 the following were used:

a, b, c, d, e	$1/s$ to $1/s^5$	coefficients of the frequency equations
a', b', c', d'	$1/s$ to $1/s^4$	coefficients of the frequency equations
D_1 to D_5	$1/s$ to $1/s^{15}$	Hurwitz determinants
D_x, D_z		damping of oscillations in roll and yaw
f_x, f_z	cps	frequency of the oscillations in roll and yaw
$Fr = \frac{v^2}{bg}$	–	Froude number
H	km	altitude
k	–	control-gearing ratio
m	–	measuring ratio (cf. p. 7)
$l_{...}$	–	moment coefficients of the lateral stability (cf. pp. 12 and 13)
$n_{...}$	–	moment coefficients of the lateral stability (cf. pp. 12 and 13)
$T_F = \frac{2G}{g\rho vF}$	s	aerodynamic time unit
v_o	m/s	equilibrium flight velocity
z	–	variables of the frequency equation
λ	1/s	variables of the frequency equation
κ	–	rotational angle about the vertical axis (yaw angle)
μ_s	–	relative density of the mass of the airplane
Π, π	$1/s^2$	products of the pairs of roots of the frequency equation
Σ, σ	1/s	sums of the pairs of roots of the frequency equation

$\tau = \frac{t}{T_F}$ non-dimensional time

II. REASONS FOR AND AIM OF THE INVESTIGATION

For a longitudinally and laterally stable glide bomb design developed in the DFS "Ernst Udet" [1][1] two possibilities existed for obtaining a rectilinear flight path: either without automatic control, with maintenance of high manufacturing accuracy, in particular very good constructional symmetry, or else with the aid of an automatic control; in the latter case too great constructional accuracy could be foregone. Since, however, even slight constructional defects (probably unavoidable even in most careful manufacture) may lead to considerable disturbances [2], only the second possibility could be applied in practice. The longitudinal stability could be ensured very simply, (even without use of automatic control), and lateral stability, attainable only with much more difficulty, became the main object; it was to be attained with the aid of a special control arrangement in the glide bomb. In order to limit to a minimum the drop tests which are expensive and require much time, the lateral-stability properties of the bomb and the most favorable design of the automatic pilot and the most favorable adjustment of the control possible within practical limits were to be clarified by calculation beforehand. This is, essentially, the aim of the present report.

III. DESCRIPTION OF THE AUTOMATICALLY-CONTROLLED GLIDE BOMB

Figure 1 is an outline drawing of the investigated glide bomb. The fuselage consists of a bomb SC-500, the cylindrical part of which was slightly elongated; the short blunt tail section was replaced by a longer one of aerodynamically more favorable design, which was provided with a vertical fin and housed the control. The two wing halves, provided with ailerons, are attached laterally at the center of the fuselage. The wing shows pronounced sweep-back but not dihedral and is at zero incidence with respect to the fuselage. Since the wing section is symmetrical, both wing halves are of equal area. For attainment of a sufficient longitudinal stability and adherence to a certain c_a-value an appropriate position of the center of gravity is selected and, since

[1]Numbers in brackets refer to the references at the end of the report.

tail surfaces are lacking, both control surfaces are accordingly trimmed upwards. This then is the zero position about which they are differentially actuated to serve as ailerons. They may also be simultaneously deflected through a differential linkage as elevators. Chiefly for variation of the yawing moment due to sideslip (weathercock stability) a stabilizing fin was sometimes attached to the bomb nose; it slightly influenced, among other factors, the lateral force, the rolling moment due to sideslip, and the damping in yaw.

The control required for attaining rectilinear flight and sufficient lateral stability was a gyro control developed in the DFS "Ernst Udet" which operates without time lag, that is, effects any control-surface deflection pertaining to an angular displacement without delay. No device of any kind producing a leading of the control-surface deflection relative to the angular displacement was applied. Of course, any other control could be used provided it operated according to the same control law so that the results of this investigation apply to it, too.

The control gyroscope is installed so that its measuring axis lies in the symmetry plane of the glide bomb and forms a tilt angle with the vertical axis; the purpose is sensitive measurement of banking and yawing deviations and their transformations into an aileron deflection through a gear with adjustable ratio. The proportionality between bank and yaw and aileron deflection is influenced by the gyroscope tilt angle. The following control method results:

If, due to any disturbance, only a bank of the glide bomb occurs, a proportional aileron deflection counteracts it and after a few damped oscillations the "undisturbed-flight-path" position is re-established. An error occurring in the flight path[2] also is measured by the control; the originating aileron deflection produces a bank toward the side opposite to the course deviation. Thereby the flight path and with it (due to its weathercock stability) the glide bomb are, as intended, turned back in the direction of the undisturbed flight path (azimuth stability). With the approach toward the undisturbed flight path the bank is again neutralized so that an aperiodical transitional motion in the direction of the undisturbed flight path results which could, with reference to Mathias [3], be designated as "yawing" motion; damped rolling and yawing oscillations are superimposed on it.

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[2]In case of calm air, this will apply throughout.

IV. SETTING UP OF THE FREQUENCY EQUATION

(CHARACTERISTIC EQUATION)

a. Preliminary Remarks

For safety reasons a release velocity far below the equilibrium velocity v_o of the glide bomb was selected for releasing the glide bomb from the carrier plane so that in no case would an immediate ascent of the flight path occur. However, this leads to a path oscillation (phugoid) of the glide bomb; without suitable control measures it is improperly damped and has a relatively long period of oscillation so that during the considered time interval from the release of the glide bomb onward a steady flight path does not exist. Thus, investigations regarding the lateral stability of a glide bomb are of increased practical importance when they take the simultaneous path oscillation into account; this will be attempted below.

The numerical integration of the known six differential equations of the longitudinal and lateral motion, with consideration of the control law and with inclusion of terms of higher order, was not intended. Such a numerical integration would have required an intolerable calculating expenditure and still would have given results in limitedly accurate agreement with actual conditions, due to estimated aerodynamic coefficients of the glide bomb and to other simplifications. Accordingly a few approximations (suggested by the nature of the problem) were made which led to far-reaching simplifications of the calculation and to a no longer intolerable expenditure of work for the numerical evaluation.

Of special importance is the assumption that c_a, in this investigation practically dependent on α only, remains constant during the flight of the glide bomb. This assumption is justified since the relatively properly damped α-oscillation is considerably more rapid than the path oscillation, thus is damped quickly, and α then retains, without change, approximately its equilibrium value corresponding to the static longitudinal stability. Besides, this behavior – estimated by rough calculation – was well confirmed by film recordings of the releases. Hence all coefficients which are functions of α (or c_a) remain constant and lead to an extensive simplification of the calculation.

Furthermore it is presupposed, as is customary, that the course of the longitudinal motion is practically independent of the lateral motion. This is justified here for the added reason that the coefficients determining the longitudinal motion are, in the first approximation, independent of the unsymmetrical state of flight of the lateral motion. This assumption is justifiable only as long as one stays within the limits of normal disturbances, the only ones considered here.

The opposite, that the longitudinal motion has no significant effect on the lateral motion, no longer holds true here. This applies only when a generally steady and little disturbed flight path is being considered. Since, however, the deviations of the longitudinal motion from a steady flight path occurring in the present case far exceed the magnitude of small disturbances, they do influence the lateral motion quite noticeably and must be taken into account. In the complete equations of the lateral motion [4] there appear in connection with the longitudinal motion, aside from the constant α, the values of v, γ, and the first derivative with respect to time, $\dot{\gamma}$. It is true, these three values vary, but only with a frequency up to two orders of magnitude lower than that of the rolling and yawing oscillation.

If one, therefore, considers a small part of the course of the motion, sufficient for judging the stability of the lateral motion, after a disturbance, v, γ, and $\dot{\gamma}$ vary so little in the meantime that they may be regarded as practically constant.

This assumption enables solution of the three differential equations of the lateral motion in the conventional manner without considerable difficulties. If one wants to find out whether lateral stability prevails during the entire course of the longitudinal motion, one subdivides the course of motion appropriately and then introduces the corresponding values of v, γ, and $\dot{\gamma}$ as sectionally invariable into the lateral motion. In the present investigation the lateral stability determined at the four points denoted in figure 2 was regarded as sufficient for the stability of the lateral motion for the full duration of a path oscillation and thus during the entire free flight of the glide bomb.

In order to include with certainty even the largest path oscillations to be expected at the glide-bomb releases, the release velocity was assumed to be only 60 percent of the equilibrium velocity v_o of the glide bomb.

The lateral stability for controlled longitudinal motion also may be investigated in the manner described above, although only under the limiting presuppositions that α remains practically constant or at least sectionally almost constant and that the oscillation period of the path oscillation considerably exceeds that of the rolling and yawing oscillation.

Furthermore the influence of the Mach number is neglected. Although a velocity of more than 200 meters per second, which corresponds to a Mach number of over 0.6, was attained in drop tests and though the present investigation is intended to include these velocities, a consideration of the influence of the Mach number on the coefficients (which is known only very inadequately) would be so uncertain that we desisted from it. Besides, as tests by Göthert [5] and [6] show, really critical conditions start appearing only at Mach numbers around 0.7; here, then, lies the limit of validity for the present investigation.

Finally it should be mentioned that a change of the air density in the course of the lateral motion is not taken into consideration: first, the altitude of a single initial disturbance causing the oscillation (which is decisive for the determination of the lateral stability) does not vary so greatly as to render that profitable; second, the (after all, significant) difference in altitude between peak and bottom of the path oscillation (where the lateral stability is calculated) is included in the investigation of the influence of relative aircraft mass densities μ_s of different magnitude.

Before we start setting up the equations for the lateral motion we shall discuss the control law of the built-in control since it is being introduced into these equations.

b. The Control Law

As described on page 4, angles of rotation about the longitudinal as well as about the vertical axis are measured by means of the control used here, and a rolling moment proportional to them was produced through an aileron deflection without delay. However, since an aileron deflection generally also causes a yawing moment, this latter must be taken into account as well. The additionally possible slight variations of lift, drag, and lateral force are neglected since the aileron deflection causing them remains small due to a disturbance assumedly small throughout. If the angle of rotation about the vertical axis to be designated as yaw angle is (for want of more suitable letters) denoted by κ and is counted starting from the desired position, the control law reads:

$$\xi = k(\kappa + m\varphi)$$

Therein k signifies the control-gearing ratio if one understands it as the ratio of the aileron deflection to the yaw angle with simultaneous insensitivity to bank,* and m, the measuring ratio indicating what fraction of a bank is measured by the control like a yaw angle and transformed into an aileron deflection.

*Translator's note: Literally, "error-free bank."

In the arrangement selected here m depends on the degree of tilt of the control gyroscope in the glide bomb and remains throughout smaller than one, that is, the bank is always measured with less sensitivity than the yaw angle.

Since later on, the rolling and yawing moments stemming from the control also are needed, the expressions for those are set up as well. Both moments are dependent only on κ and φ so that one may express them as follows:

$$\text{aileron rolling moment} = L_\kappa{}'\kappa + L_\varphi{}'\varphi$$

$$\text{aileron yawing moment} = N_\kappa{}'\kappa + N_\varphi{}'\varphi$$

The moment essential for the lateral stability of the controlled glide bomb, newly added in comparison to the uncontrolled condition, is L_φ. It signifies an immediately effective restoring moment for the longitudinal axis which intervenes directly in case of banking errors. This restoring ability about the longitudinal axis corresponds to the yawing moment due to sideslip which is important for the vertical axis and is regarded later in the same way as likewise characteristic variable. Taking the control law into account, one finds $L_\varphi{}' = kmL_\xi{}'$ which shows that not merely the control-gearing ratio, measuring ratio, or aileron effectiveness are of importance, but their combined action.

The moment $L_\kappa{}'$ coming into effect in case of lateral deviations serves, as described on page 4, only for maintaining the course and thus is less important than L_φ for the lateral stability of the glide bomb.

The aileron yawing moment originating in case of an aileron actuation is produced only unintentionally. It does have a slight influence on the lateral stability as will be seen later; thus it must, as a precaution, be taken into consideration, but is otherwise of no importance.

After these preliminary remarks the dynamic relations of the lateral motion are set up. The longitudinal and vertical axes of the glide bomb were selected as the two axes about which equilibrium of moments must prevail; for the equilibrium of forces a horizontal direction of reference perpendicular to the flight path was chosen. The following section (c) offers the reasons for this latter choice which is at variance with convention.

c. The Horizontal Forces Perpendicular to the Flight Path

Basically, any direction of reference would be equally suitable for the equilibrium of forces if it only contains components of the forces responsible for the lateral path curvature. Preference is to be given to the one which appears especially illustrative from the dynamical viewpoint.

According to what was said on page 6, the longitudinal inclination of the flight path γ varies so little during the brief time interval sufficient for consideration of the lateral motion that it may be regarded as invariable. Thus the path section considered is part of a helix which one may visualize as wound on a vertical circular cylinder of the radius of the flight path, as shown in figure 3.

On this helical line travels the glide bomb which for the consideration with respect to forces of the lateral path equilibrium must be visualized as a mass point; the position of the body axis of the glide bomb is, for the time being, unimportant. The direction of the centripetal force which causes the lateral curvature of the flight path and coincides with the principal normal of the helix is horizontal and perpendicular to the flight path. Thus it is logical to balance the components of the air forces in this selected direction with the mass force which is effective as centrifugal force. All forces acting in the direction of flight path and gravity have no influence on the lateral curvature of the flight path visualized as part of a helix, because they are perpendicular to its principal normal.

In the specifications DIN L 100 this axis used as the line of attack for the lateral-force equilibrium is not especially characterized; however, it coincides with y_g in case one puts $\chi = 0$. Actually $(y_g)_{\chi=0}$ is there perpendicular to z_g as well as to x_a and the cosines of the angles formed with the effective air forces may be taken directly from the attached form. Since the total air force is mostly given by its components A, W, and Q and since W, lying in the flight path x_a, does not yield a contribution, the equilibrium ratio becomes:

$$A \cos(-z_a, y_g)_{\chi=0} + Q \cos(y_a, y_g)_{\chi=0} = \frac{G}{g} v(\cos \gamma) \dot{\chi}$$

Additionally, it is here tacitly assumed that the laterally effective air forces – those stemming from the rotations of the glide bomb about its longitudinal and vertical axis as well as from the aileron deflection – are so small that they may be neglected [3] and [7].

With $A = c_a \frac{\rho}{2} v^2 F$ and $Q = c_q' \frac{\rho}{2} v^2 F\beta$ and special consideration of the fact that G here must not be replaced by $\frac{c_a}{\cos \gamma} \frac{\rho}{2} v^2 F$ as is customary in case of steady rectilinear flight, one then obtains

$$c_a \frac{\rho}{2} v^2 F \sin \mu + c_q' \frac{\rho}{2} v^2 F\beta \cos \mu = \frac{G}{g} v \cos \gamma \dot{\chi}$$

Since one has to deal for the lateral motion (however, not for the longitudinal motion!) only with small disturbances, one may equate $\sin \mu = \mu$ and $\cos \mu = 1$ and there results

$$c_a \frac{\rho}{2} v^2 F \mu + c_q' \frac{\rho}{2} v^2 F\beta = \frac{G}{g} v \cos \gamma \dot{\chi} \tag{1}$$

d. The Moments about the Longitudinal Axis

Here the complete Euler equation must be stated in order to make consideration of the influence of the gyroscopic terms possible:

$$L = J_x \dot{\omega}_x + (J_z - J_y)\omega_y \omega_z$$

L contains all rolling moments stemming from the air forces; they are caused by sideslip, rolling and yawing, and by the aileron deflection.

Since for glide bombs with relatively very small span the moments of inertia J_y and J_z are almost equal, the slight difference between them, multiplied by the small rotational velocity ω_y stemming from the longitudinal motion and the yawing velocity, is so insignificant, especially in comparison with the strong aileron moment, that the right-hand gyroscopic term is neglected. Thus there remains after introduction of all partial moments

$$\underbrace{L_\beta'\beta}_{\text{sideslip}} + \underbrace{L_x'\omega_x}_{\text{rolling}} + \underbrace{L_z'\omega_z}_{\text{yawing}} + \underbrace{L_\varphi'\varphi + L_\kappa'\kappa}_{\text{control}} = J_x \dot{\omega}_x$$

e. The Moments about the Vertical Axis

Euler's equation here reads

$$N = J_z\dot{\omega}_z + \left(J_y - J_x\right)\omega_x\omega_y$$

Due to the very small span of the glide bomb J_x is by about one order of magnitude smaller than J_y, and the expression $\left(J_y - J_x\right)$ may be replaced by J_z. N is subdivided as in (d) and one obtains

$$\underbrace{N_\beta{}'\beta}_{\text{sideslip}} + \underbrace{N_x{}'\omega_x}_{\text{rolling}} + \underbrace{N_z{}'\omega_z}_{\text{yawing}} + \underbrace{N_\varphi{}'\varphi + N_\kappa{}'\kappa}_{\text{control}} = J_z\left(\dot{\omega}_z + \omega_x\omega_y\right) \tag{3}$$

Here the gyroscopic term must not be neglected; even though it is not important, it is, compared to the yawing moments due to the air forces, sufficiently significant to be taken into consideration, in order to avoid errors.

There is still another reason for the seemingly different evaluation of the gyroscopic terms in the moment equations for longitudinal and vertical axis: The sum of the moments about the longitudinal axis is dominated by the aileron moment which acts on a large lever arm of about half the span and always may be made the predominant rolling moment by suitable design of the control and ailerons. Conditions are different for the yawing moments; there the air force yielding the restoring moment is not so widely variable as for the aileron, due to the spatial limitation of the vertical fin; furthermore, it acts on a lever arm which measures only centimeters and is, moreover, relatively uncertain due to manufacturing inaccuracies occurring in practice, caused either by the position of the center of gravity or other faults in construction. For this reason the moments effective about the vertical axis (which is more sensitive) must be taken into consideration to a much higher degree than is required for the longitudinal axis, if surprises are to be avoided.

f. The Solution of the Three Equations of Motion

The equations of motion, here compiled once more

$$c_a \frac{\rho}{2} v^2F\mu + c_q{}' \frac{\rho}{2} v^2F\beta = \frac{G}{g} v \cos\gamma\,\dot{\chi} \tag{1}$$

$$L_\beta{}'\beta + L_x{}'\omega_x + L_z{}'\omega_z + L_\varphi{}'\varphi + L_\kappa{}'\kappa = J_x\dot{\omega}_x \tag{2}$$

$$N_\beta'\beta + N_x'\omega_x + N_z'\omega_z + N_\varphi'\varphi + N_\kappa'\kappa = J_z\left(\dot{\omega}_z + \omega_x\omega_y\right) \tag{3}$$

are three simultaneous differential equations; however, their unknowns are not yet uniform. In itself, it would be a matter of indifference which one is selected for the solution; however, since the first equation is particularly characteristic for the lateral motion, the three unknowns μ, β, and χ appearing in that equation are to be maintained throughout.

Thus all variables appearing in the second and third equation must be expressed in terms of μ, β, and χ and their derivatives with respect to time. This is done partly according to Rautenberg [8] and [9], with the presupposition that the disturbances of the lateral motion are small (thus the three unknowns just mentioned are only a little different from zero) and with the presupposition that for the longitudinal motion α and $\dot{\gamma}$ — but no longer γ — may be regarded as small quantities. Since steep nose dives may be left out of consideration anyhow, the following approximations, valid up to about $\gamma = -45^\circ$, result:

$$\varphi = \mu + \beta \tan \gamma \qquad \omega_x = \dot{\mu} - \dot{\chi} \sin \gamma$$

$$\omega_y = \dot{\gamma}$$

$$\kappa = \beta + \chi \cos \gamma \qquad \omega_z = \dot{\beta} + \dot{\chi} \cos \gamma$$

Furthermore, the moment increases L' and N', divided by the respective moments of inertia J_x and J_z, are expressed by the following non-dimensional moment coefficients of lateral stability:

damping in roll $$l_x = -\left[\frac{s}{i_x}\right]^2 \frac{\partial c_L}{\partial \omega_x} \frac{v}{s}$$

rolling moment in yawing $$l_z = \left[\frac{s}{i_x}\right]^2 \frac{\partial c_L}{\partial \omega_z} \frac{v}{s}$$

rolling moment due to sideslip $$l_\beta = \left[\frac{s}{i_x}\right]^2 \frac{\partial c_L}{\partial \beta}$$

aileron rolling moment (total) $$l_\xi = \left[\frac{s}{i_x}\right]^2 \frac{\partial c_L}{\partial \xi}$$

aileron rolling moment (due to φ) abbreviatedly: aileron moment $$l_\varphi = -\left[\frac{s}{i_x}\right]^2 \frac{\partial c_L}{\partial \varphi}$$

aileron rolling moment (due to κ) $$l_\kappa = \left[\frac{s}{i_x}\right]^2 \frac{\partial c_L}{\partial \kappa}$$

rolling-yawing moment $$n_x = \left[\frac{s}{i_z}\right]^2 \frac{\partial c_N}{\partial \omega_x} \frac{v}{s}$$

damping-in-yaw $$n_z = -\left[\frac{s}{i_z}\right]^2 \frac{\partial c_N}{\partial \omega_z} \frac{v}{s}$$

yawing moment due to sideslip (weathercock stability) $$n_\beta = -\left[\frac{s}{i_z}\right]^2 \frac{\partial c_N}{\partial \beta}$$

aileron yawing moment (total) $$n_\xi = \left[\frac{s}{i_z}\right]^2 \frac{\partial c_N}{\partial \xi}$$

aileron yawing moment (due to φ) $$n_\varphi = \left[\frac{s}{i_z}\right]^2 \frac{\partial c_N}{\partial \varphi}$$

aileron yawing moment (due to κ) $$n_\kappa = \left[\frac{s}{i_z}\right]^2 \frac{\partial c_N}{\partial \kappa}$$

For the four moment coefficients which are decisive for the restoring and damping moments about the longitudinal and vertical axis the sign was selected so that they appear – as generally customary in the theory of oscillations – as positive quantities.

Taking the control law valid here $\xi = k(\kappa + m\varphi)$ into consideration, some moment coefficients may be transformed, after brief intermediary calculation, as follows:

$$l_\varphi = -kml_\xi \qquad n_\varphi = -\frac{n_\xi}{l_\xi} l_\varphi$$

$$l_\kappa = -\frac{1}{m} l_\varphi \qquad n_\kappa = -\frac{1}{m}\frac{n_\xi}{l_\xi} l_\varphi$$

By these transformations it was attained that the connection of the aileron moment l_φ which is particularly important for the lateral stability with the known values of the control (k,m) and the automatic pilot (l_ξ) becomes manifest and that the three remaining moment coefficients are expressed by this aileron moment l_φ and a further known numerical value $\left[\frac{n_\xi}{l_\xi}\right]$.

If, furthermore, the aerodynamic time unit $T_F = \frac{2G}{g\rho vF}\left[s = \text{Dimension}\right]$ and the relative aircraft mass density $\mu_s = \frac{2G}{g\rho sF}$ are introduced, there results finally:

	Dimension		Dimension
$\frac{L_x'}{J_x} = -\frac{1}{T_F} l_x$	1/s	$\frac{N_x'}{J_z} = \frac{1}{T_F} n_x$	1/s
$\frac{L_z'}{J_x} = \frac{1}{T_F} l_z$	1/s	$\frac{N_z'}{J_z} = -\frac{1}{T_F} n_z$	1/s
$\frac{L_\beta'}{J_x} = \frac{\mu_s}{T_F^2} l_\beta$	$1/s^2$	$\frac{N_\beta'}{J_z} = -\frac{\mu_s}{T_F^2} n_\beta$	$1/s^2$
$\frac{L_\varphi'}{J_x} = -\frac{\mu_s}{T_F^2} l_\varphi$	$1/s^2$	$\frac{N_\varphi'}{J_z} = -\frac{n_\xi}{l_\xi}\frac{\mu_s}{T_F^2} l_\varphi$	$1/s^2$
$\frac{L_\kappa'}{J_x} = -\frac{1}{m}\frac{\mu_s}{T_F^2} l_\varphi$	$1/s^2$	$\frac{N_\kappa'}{J_z} = -\frac{1}{m}\frac{n_\xi}{l_\xi}\frac{\mu_s}{T_F^2} l_\varphi$	$1/s^2$

Therewith the equations (1), (2), and (3), on pp. 11 and 12, arranged according to the three unknowns, μ, β, and χ, assume the following form:

$$c_a\mu + c_q'\beta - T_F \cos\gamma\dot{\chi} = 0 \tag{1a}$$

$$- \frac{\mu_s}{T_F{}^2} l_\varphi \mu - \frac{1}{T_F} l_x \dot{\mu} - \ddot{\mu}$$

$$+ \frac{\mu_s}{T_F{}^2} \left[l_\beta - \left(\frac{1}{m} + \tan \gamma \right) l_\varphi \right] \beta + \frac{1}{T_F} l_z \dot{\beta}$$

$$- \frac{1}{m} \frac{\mu_s}{T_F{}^2} l_\varphi (\cos \gamma) \chi + \frac{1}{T_F} \left(l_x \sin \gamma + l_z \cos \gamma \right) \dot{\chi}$$

$$+ (\sin \gamma) \ddot{\chi} = 0 \tag{2a}$$

$$- \frac{n_\xi}{l_\xi} \frac{\mu_s}{T_F{}^2} l_\varphi \mu + \frac{1}{T_F} \left(n_x - T_F \dot{\gamma} \right) \dot{\mu}$$

$$- \frac{\mu_s}{T_F{}^2} \left[n_\beta + \frac{n_\xi}{l_\xi} \left(\frac{1}{m} + \tan \gamma \right) l_\varphi \right] \beta - \frac{1}{T_F} n_z \dot{\beta} - \ddot{\beta}$$

$$- \frac{1}{m} \frac{n_\xi}{l_\xi} \frac{\mu_s}{T_F{}^2} l_\varphi (\cos \gamma) \chi$$

$$- \frac{1}{T_F} \left[\left(n_x - T_F \dot{\gamma} \right) \sin \gamma + n_z \cos \gamma \right] \dot{\chi} - (\cos \gamma) \ddot{\chi} = 0 \tag{3a}$$

The further treatment of these three equations does not offer any additional peculiarities; the unknowns, μ, β, and χ, are put proportional to $e^{\lambda t}$ and λ is then determined from the condition that the principal determinant of the previous equation system must vanish if the solutions are not to be identically zero. One then obtains for λ an equation of the fifth degree, the characteristic equation:

$$\lambda^5 + a\lambda^4 + b\lambda^3 + c\lambda^2 + d\lambda + e = 0$$

with the coefficients

$$a = \frac{1}{T_F}\left(l_x + n_z + c_q{}' - c_a \tan\gamma\right)$$

$$\begin{aligned} b = \frac{\mu_s}{T_F{}^2}\Bigg\{&\left[\frac{n_\xi}{l_\xi}\left(\frac{1}{m} + \tan\gamma\right) + 1\right] l_\varphi + n_\beta \\ &+ \frac{1}{\mu_s}\left[l_x n_z - l_z\left(n_x - T_F\dot{\gamma}\right) + \left(c_q{}' - c_a \tan\gamma\right)\left(l_x + n_z\right)\right]\Bigg\} \end{aligned}$$

$$\begin{aligned} c = \frac{\mu_s}{T_F{}^3}\Bigg(\Bigg\{&\frac{n_\xi}{l_\xi}\left[l_x\left(\frac{1}{m} + \tan\gamma\right) + l_z\right] + \left(n_x - T_F\dot{\gamma}\right)\left(\frac{1}{m} + \tan\gamma\right) \\ &+ n_z + \left(c_q{}' - c_a \tan\gamma\right)\left[\frac{n_\xi}{l_\xi}\left(\frac{1}{m} + \tan\gamma\right) + 1\right]\Bigg\} l_\varphi \\ &+ l_x n_\beta - l_\beta\left(n_x - T_F\dot{\gamma}\right) + c_a\left(l_\beta - n_\beta \tan\gamma\right) \\ &+ \frac{1}{\mu_s}\left(c_q{}' - c_a \tan\gamma\right)\left[l_x n_z - l_z\left(n_x - T_F\dot{\gamma}\right)\right]\Bigg) \end{aligned}$$

$$\begin{aligned} d = \frac{\mu_s}{T_F{}^4}\Bigg[&\mu_s\left(\frac{n_\xi}{l_\xi} l_\beta + n_\beta\right) l_\varphi \\ &+ \left(c_q{}' - c_a \tan\gamma\right)\Bigg\{\frac{n_\xi}{l_\xi}\left[l_x\left(\frac{1}{m} + \tan\gamma\right) + l_z\right] \\ &+ \left(n_x - T_F\dot{\gamma}\right)\left(\frac{1}{m} + \tan\gamma\right) + n_z\Bigg\} l_\varphi \\ &+ c_a\left\{l_\beta\left[n_z + \left(n_x - T_F\dot{\gamma}\right)\tan\gamma\right] - \left(l_z + l_x \tan\gamma\right) n_\beta\right\} \end{aligned}$$

$$e = \frac{\mu_s{}^2}{T_F{}^5}\,\frac{c_a}{m}\left(\frac{n_\xi}{l_\xi} l_\beta + n_\beta\right) l_\varphi$$

Compared to the frequency equation of the uncontrolled lateral motion which is of the fourth degree, its degree is now increased by one. The reason lies in the fact that for the controlled glide bomb treated here, due to the control a formerly non-existent tie-up with a direction in space or, more accurately, with a directional plane, is added.

λ was not made non-dimensional by means of the aerodynamic time unit T_F because here that would not offer any advantage; it would not make any difference for the later representation of stability domains, but would be of disadvantage for the determination of the oscillation frequencies since the result has to be converted again to seconds afterwards.

No difficulty arises if, nevertheless, a non-dimensional representation of the frequency equation should be required for some reason. One has to take into consideration that in solving for the three unknowns μ, β, and χ one should have made them proportional, not to $e^{\lambda t}$, but to $e^{z\tau}$, with the non-dimensional z appearing in the frequency equation instead of λ and τ in contrast to t, denoting a non-dimensional time. Since, therefore, $z\tau$ must be equal to λt and, after selection of an (at first arbitrary) time unit T, the non-dimensional time becomes $\tau = \frac{t}{T}$, there results $\lambda = \frac{z}{T}$. If this value is inserted into the frequency equation and the total equation is, moreover, enlarged by T^5, it can be seen easily that due to the specific selection of the aerodynamic T_F for T the T_F powers in parentheses for all coefficients cancel each other; one obtains the non-dimensional form of the frequency equation

$$z^5 + aT_F z^4 + bT_F^2 z^3 + cT_F^3 z^2 + dT_F^4 z + eT_F^5 = 0$$

It must be well considered here that – because of the nonsteady longitudinal motion due to a gyroscopic term – the T_F in the coefficients bT_F^2, cT_F^3 and dT_F^4 remains and thus in the non-dimensional frequency equation also the direct influence of the velocity is not cancelled.

g. Brief Discussion of the Coefficients of the Frequency Equation (Characteristic Equation)

As will be shown later (p. 20), one condition required for attainment of lateral stability is that all coefficients be positive. If a few numerical values, enumerated on pp. 25 and 26, are anticipated, one obtains for the separate coefficients under this point of view the following results:

a is with certainty always positive since the chiefly important dampings l_x and n_z about the longitudinal and vertical axes are positive as is also the lateral-force increase and, in descending flight, $-c_a \tan \gamma$. Besides, this last expression equals c_w for steady rectilinear flight, but not under any other conditions.

b also is positive because the restoring moments l_φ and n_β predominate for the longitudinal and vertical axis; due to the high relative aircraft mass density μ_s here – and also for the other coefficients – chiefly the terms containing that quantity are of importance.

c depends on the expressions $l_\varphi n_z$ and $l_x n_\beta$ and thus must always be made positive.

d depends to a high degree on $\left[\frac{n_\xi}{l_\xi} l_\beta + n_\beta\right]$, thus on the sign of e.

e will be transformed somewhat further; using the expression for n_φ on p. 13 one obtains

$$\left[\frac{n_\xi}{l_\xi} l_\beta + n_\beta\right] l_\varphi = l_\varphi n_\beta - l_\beta n_\varphi$$

Similarly to the static lateral stability of uncontrolled flight this expression, too, has special significance. If one visualizes that the glide bomb sideslips to the side opposite to the bank during an error in banking, it is the rolling moment l_φ stemming from the aileron deflection and the weathercock stability n_β which attempts to right the glide bomb again whereas the rolling moment due to sideslip l_β and the aileron yawing moment n_φ tend to increase the bomb's angular deviation. Thus sufficiently large values of l_φ and n_β must be selected in order to have e with certainty turn out positive, particularly if n_φ is known only approximately or the sign varies (in case of aileron deflections of different magnitudes), as happens occasionally.

This brief discussion, which has shown that all coefficients of the frequency equation always may be made positive, will have to suffice for the time being; detailed treatment of the connections with the dynamic lateral stability will follow in later paragraphs.

V. THE EVALUATION OF THE FREQUENCY EQUATION

(CHARACTERISTIC EQUATION)

a. The Method Followed

The frequency equation and knowledge of the values contained in it offer a means for making, in any case, a statement regarding the stability of the glide bomb after a disturbance of the lateral motion, and for calculating the occurring frequencies and dampings. This approach by particular values is practically the only one, if a comprehensive view of more general validity is to be given, as this investigation aims to do; a general solution is not possible and clear fundamental insight into the complicated inter-relationship of the variables of this problem is hardly ever obtainable. Thus it is the only possible course to strive for a comprehensive survey of the influence of the frequency equation on the lateral stability by numerical substitution of all variable quantities in automatic pilot and control and by the evaluation of the frequency equation.

In limiting this investigation to an automatic pilot of the type used in the glide bomb here described, part of the values in the coefficients a to e (p. 16) have been made practically constant. These values will probably not essentially deviate for automatic pilots of slightly different form; the larger parts of the values, however, may be varied or are known only rather uncertainly so that their influence must be taken into consideration. If, moreover, all possible combinations should be formed, such an enormous number of cases to be calculated would result that one will try to attain the goal by a less tiresome method.

Starting from a principal case, characterized on the whole by mean values of the variables concerned, the influence of these quantities on the lateral stability was investigated one at a time. One forms – if the expression be permitted – the partial differential quotients of the influence on the lateral stability with respect to all possible variations of the controlled automatic pilot. In this way one obtains a complete picture of in what respects alterations on the glide bomb must be made in order to improve its lateral stability. Of course, there is no guarantee that for a different selection of the principal case the influences would have made themselves felt in the same way. It is even feasible that an influence, favorable for a certain case, might turn into the opposite for another. However, where that is the case, one will always spot-check anyway, in order to avoid the minor inaccuracies of this method. At any rate, the expenditure in calculation for this method is the minimum imaginable for a clarification of this rather involved problem (with the objective set up initially in mind); thus this method was chosen.

b. The Stability Domain

For visualization of the numerical results to be discussed later, a form of representation was selected which is also customary in the treatment of the lateral stability of uncontrolled airplanes. There in a l_β, n_β-plane the domains are plotted where, with the remaining airplane values otherwise retained, lateral stability or instability prevails. These domains are bounded by curves, the course of which shifts if one of these airplane values is altered and appears then as a parameter of these curves.

The value l_β (rolling moment due to sideslip) which is plotted on one axis and has for the uncontrolled airplane the significance of an indirect restoring force about the longitudinal axis, is like n_β (the yawing moment due to sideslip, or weathercock stability) in general controllable within wide limits, and is particularly important for the lateral stability. For the controlled glide bomb of this investigation l_β is replaced by l_φ (aileron moment) which is a directly effective restoring moment about the longitudinal axis and thus becomes comparable to n_β. Thus the l_φ, n_β-plane logically suggests itself for representation of the regions where stability or instability of the lateral motion prevails.

In order to determine whether at any point of the l_φ, n_β-plane stability prevails and, therefore, the disturbance of the lateral motion shows damping, one must investigate whether all roots λ of the frequency equation have a negative real part, and whether consequently the frequency equation is a Hurwitz equation. This is the case when the 5 Hurwitz determinants

$$D_1 = a$$

$$D_2 = ab - c$$

$$^{*}D_3 = (ab - c)c - (ad - e)a$$

$$D_4 = (ab - c)(cd - be) - (ad - e)^2$$

$$D_5 = eD_4$$

formed from the coefficients of the frequency equation of the fifth degree

$$\lambda^5 + a\lambda^4 + b\lambda^3 + c\lambda^2 + d\lambda + e = 0$$

are all positive.

*NACA Reviewer's note: D_3 as presented is as in the German text. This reviewer believes, however, that in this determinant $e = 0$ and should be used as such.

Inversely, if all roots λ of the frequency equation have a negative real part, all values D_1 to D_5 are positive. It is a consequence of this criterion (and completely equivalent to it) that all coefficients a to e, furthermore D_2 and D_4, turn out positive.

If one now visualizes the point in the l_φ,n_β-plane considered before (which assumedly has been found to be stable) as moving, the real parts of the roots, the coefficients a to e, and the 5 Hurwitz determinants change. If one now comes to a point where, for instance, the real part of a real root – and thus this root itself – vanishes, e too vanishes, since $|e|$ is proportional to the magnitudes of all roots; on the other hand, if one reaches a point where the real part of a complex root is zero, D_4 becomes zero, as can easily be shown. In any case, however, $D_5 = eD_4$ vanishes, and since for reasons of continuity the totality of all points in the l_φ,n_β-plane where stability prevails must be connected, the following important statement is valid:

The limit of the stability domain lies at $D_5 = 0$.

Thus it is sufficient for graphic representation of the stability domain to plot $D_5 = 0$, or, simpler, $e = 0$ and $D_4 = 0$ as limiting lines of a region at an arbitrary point of which stability is known to prevail, as illustrated in figure 4.

c. Frequencies and Dampings

In addition to the determination of lateral stability, it is often important to know what frequencies and dampings occur when a disturbance is damped. Thus one investigates the magnitude of the roots λ of the frequency equation for a certain point of the l_φ,n_β-plane.

At least one root is always real and may be numerically determined without difficulties. In order to procure an approximate value for a root, one considers the problem itself and attempts to make the essentials of the prescribed motion after a disturbance stand out under simplifying assumptions, and thus to obtain an indication for an approximate solution.

According to p. 4, the turning of the flight path into its undisturbed direction takes place as an aperiodic transitory motion (yawing motion)(fig. 5) on which damped rolling and yawing oscillations are superimposed.

Of course, this applies only under the condition that the restoring moments about the longitudinal and vertical axes are so large that really oscillations, not aperiodic motions, originate. Then it will be possible to replace their influence on the yawing motion by the influence of a mean position of the glide bomb about which the rolling and yawing oscillations are performed. By the isolation of these two types of oscillations one arrives, therefore, at a consideration of the yawing motion alone; for obtaining the approximate solution one has only to express the dynamic relations corresponding to this train of thought.

A motion about a position of equilibrium vanishes – as can easily be seen from the differential equation of a free oscillation – if the moment of mass inertia and the damping moment vanish, or if the restoring moment by far predominates over all other moments. Under the first assumption and further neglect of the coupling moments stemming from the velocities of rotation the three equations of motion from pp. 11 and 12 then take the following form:

$$c_a \frac{\rho}{2} v^2 F\mu + c_q' \frac{\rho}{2} v^2 F\beta = \frac{G}{g} v(\cos\ \gamma)\dot{\chi}$$

$$L_\beta'\beta + L_\varphi'\varphi + L_\kappa'\kappa = L_\beta'\beta + L_\xi'\xi = 0$$

$$N_\beta'\beta + N_\varphi'\varphi + N_\kappa'\kappa = N_\beta'\beta + N_\xi'\xi = 0$$

In the second and third equation the partial moments, separately dependent on φ and κ were again comprised into a single moment caused by ξ alone. In this manner it may easily be seen that both equations can be satisfied only when β as well as ξ is permanently zero.

Of course one obtains the same result if one assumes, according to the other way of thinking, that the restoring moments by far predominate over all other moments since in that case even a vanishingly small angle of sideslip β or aileron deflection ξ produces equilibrium of moments.

Thus with $\beta = 0$ there becomes, according to p. 12, $\varphi = \mu$ and $\kappa = \chi \cos \gamma$, and according to pp. 7 and 8, the aileron deflection $\xi = k(\kappa + m\varphi) = k(\chi \cos \gamma + m\mu) = 0$ or $\mu = -\frac{\cos \gamma}{m} \chi$.

This signifies that during the yawing motion the mean bank is proportional to the error in the course of the flight path and is correspondingly neutralized when approaching the direction of the undisturbed flight path (as it was described at the beginning, p. 4, and now confirmed, somewhat more accurately, by calculation).

If this value for μ is inserted into the first equation, there results a solution of the remaining differential equation of the first order:

$$\lambda = -\frac{c_a}{mT_F}$$

This root describing the yawing motion is – as was to be expected – independent of l_φ and n_β since the magnitude of the frequency of the rolling and yawing oscillation about its mean position is, at first, of minor importance. Only in the proximity of the l_φ and n_β-axis where (as was said before) the restoring moments are so small that the presuppositions of this consideration are no longer satisfied, larger deviations from the otherwise rather accurate approximation for λ occur, as figure 7 shows for a later example.

Analogously, one may obtain indications for the frequency and damping of the remaining rolling and yawing oscillation. The rolling oscillation stands out clearly if one visualizes the disturbances about the vertical axis as completely damped; of course, the influence of the coupling moments also must be neglected. With $\beta = \chi = 0$ there remains in the equation (2a), p. 15, only the first line

$$-\frac{\mu_s}{T_F^2} l_\varphi \mu - \frac{1}{T_F} l_x \dot{\mu} - \ddot{\mu} = 0$$

From this ordinary oscillation equation then result the approximations (required for the later numerical evaluation) for the product Π and the sum Σ of the (mostly) conjugate-complex pair of roots for the rolling oscillation:

$$\Pi = \frac{\mu_s}{T_F^2} l_\varphi, \frac{1^2}{s} \quad \text{and} \quad \Sigma = -\frac{1}{T_F} l_x, \frac{1}{s}$$

Therewith the frequency f_x and the damping of the rolling oscillation become

$$f_x = \frac{1}{6.28}\sqrt{\Pi - \left(\frac{\Sigma}{2}\right)^2} \text{ cps} \quad \text{and} \quad D_x = -\frac{1}{\sqrt{\Pi}}\left(\frac{\Sigma}{2}\right)$$

Therein the damping is explained as the ratio of the damping factor of an oscillating configuration to its critical damping factor; thus it is zero for an undamped oscillation, and 1 for the aperiodic limiting case.

The way to an estimate of the yawing oscillation is perfectly analogous. If the disturbances μ and χ are now put equal to zero, one obtains from (3a), p. 15

$$-\frac{\mu_s}{T_F^2}\left[n_\beta + \frac{n_\xi}{l_\xi}\left(\frac{1}{m} + \tan\gamma\right)l_\varphi\right]\beta - \frac{1}{T_F} n_z\dot{\beta} - \ddot{\beta} = 0$$

and the product π and the sum σ of the pair of roots for the yawing oscillation are approximated:

$$\pi = \frac{\mu_s}{T_F^2}\left[n_\beta + \frac{n_\xi}{l_\xi}\left(\frac{1}{m} + \tan\gamma\right)l_\varphi\right]\frac{1}{s}^2 \quad \text{and} \quad \sigma = -\frac{1}{T_F} n_z \frac{1}{s}$$

the frequency f_z and the damping D_z of the yawing oscillation then are

$$f_z = \frac{1}{6.28}\sqrt{\pi - \left(\frac{\sigma}{2}\right)^2} \text{ cps} \quad \text{and} \quad D_z = -\frac{1}{\sqrt{\pi}}\left(\frac{\sigma}{2}\right)$$

Later results (figs. 9 and 11) show that the values Π and π which are decisive for the frequency are in good agreement with the actual values, whereas the values determining the damping Σ and σ frequently agree with them less the more one approaches the limits of the stability domain.

VI. NUMERICAL EXAMPLES

a. Numerical Data

As mentioned at the beginning, the object of this investigation is to give an insight into the lateral stability of the glide bomb treated here in order to obtain indications for a design as favorable as possible. The required numerical treatment is outlined for this special case only; however, even for attempting a quite general solution of this problem there is no other way than to use average values. The seeming limitation of general validity is a necessity due to the nature of the matter, because its inter-relations are so complicated and not readily seen. Nevertheless we shall attempt, by means of consideration of relatively extensive variations for the most essential properties of this glide bomb, to include all similar automatic pilots. However, be it here stressed once more that a comparison is possible only when the same method of control is taken as a basis.

The numerical data used stem partly from wind tunnel measurements and partly from rough calculations. Since the automatically-controlled bomb no longer has a form remotely similar to an airplane, several quantities deviate rather far from conventional ones as will be seen from the following compilation. Therein the values which form the basis for the principal case described on p. 19 are underlined; moreover, the limiting values within which the stability domains were plotted are given.

Compilation of numerical data

Lift: c_a = 0.15, 0.35, 0.55; in reference to an increase in weight (treated separately later), assuming constant velocity, further increases in lift values up to 0.643 may result.

Lateral force: the slope (partly dependent on the variations in magnitude of the front stabilizer and vertical fin) is $c_q' = \frac{\partial c_q}{\partial \beta} = 0.3$, 0.5, 0.7; the influence of the velocities of rotation ω_x and ω_z and of the aileron deflection ξ were neglected, as mentioned before on pp. 9 and 10.

The following moment coefficients of the lateral stability were obtained under the assumption that $\left(\frac{s}{i_x}\right)^2 = 15$ to 20 and $\left(\frac{s}{i_z}\right)^2 = 1.5$ to 2.5.

Damping in roll: l_x = 4, 7, 10; since it is a function of the fairly constant c_a', it can be arbitrarily varied (at the most, insignificantly, by front stabilizer and vertical fin); nevertheless the influence of a possible inaccurate estimate or measurement will be clarified by taking the two limiting values into consideration, too.

Yawing-rolling moment: l_z = 1.45, 2.45, 3.45; it varies with c_a and is, moreover, for the principal case (c_a = 0.35) altered according to the given limiting values.

Rolling moment due to sideslip: l_β = 0, 2, 4; aside from the less important influence of the front stabilizer, it may be decreased very considerably (according to oral information by A. Schieferdecker) by end plates on the wings so that the noteworthy case of a vanishing rolling moment due to sideslip is investigated as well.

Aileron rolling moment: $l_\xi = -2.6$; it is given by the ailerons and has for the controlled glide bomb not such an independent significance as the aileron moment: $l_\varphi = -kml_\xi = 0 \ldots 3$; this change for l_φ signifies for an average value of 0.2 for the measuring ratio m a variation for the control-gearing ratio k from 0 to over 5.

Rolling-yawing moment: it varies with $c_t' = \frac{\partial c_t}{\partial \alpha}$, thus is a function of $\alpha(c_a)$ and has for $c_a = 0.35$ the approximate value $n_x = -0.14$; since it appears only in the combination $(n_x - T_F\dot{\gamma})$, the influence of the gyroscopic term may be simultaneously taken into consideration in this expression by suitable selection of the limiting values for $(n_x - T_F\dot{\gamma}) = -0.279, -0.140, + 0.329$.

Damping-in-yaw: $n_z = 2, 4, 6$; the relatively small damping-in-yaw is improved by the front stabilizer and vertical fin and is decreased by negative c_t, and thus for the larger c_a-values (which has to be noted when the case arises).

Yawing moment due to sideslip (weathercock stability): $n_\beta = 0 \ldots 0.6$; it is determined by the position of the center of gravity; since the latter is mostly fixed, for reasons of longitudinal stability, this moment may be altered appropriately by front stabilizer and vertical fin.

Aileron yawing moment: its ratio to the aileron rolling moment is $n_\xi / l_\xi = -0.01, 0, +0.01$; this sometimes uncertain and fluctuating value may become important for the sign of e (as discussed on p. 18), particularly for a larger rolling moment due to sideslip, and is therefore taken into account in spite of its apparent insignificance.

An increase in the weight of the glide bomb for constant radii of inertia and an increase in flight altitude enter into the relative aircraft mass density: $\mu_s = \frac{2G}{g\rho s F} = 700, 1000, 1300$; the first value corresponds, for an altitude of $H = 1.0$ kilometer, to a weight of of $G = 600$ kg; the two other values correspond, for this weight, to flight altitudes of 4.5 and 6.9 kilometers (calculated according to the very accurate rule of thumb by Knoller [10]):

$$\rho = \frac{1}{8} \frac{20 - H}{20 + H}, \mathrm{kgs^2/m^4}$$

Aerodynamic time unit: $T_F = \mu_s \frac{s}{v} = 4.48\mathrm{s}$ for the principal case at $v = 125$ meters per second; it is, at first, not freely selectable but is a function of several values already mentioned and of the longitudinal motion of the glide bomb, and thus changes for every case individually. The same applies to the longitudinal inclination of the flight path: $\tan \gamma = -0.147$ and to the variation with time of the longitudinal inclination: $\dot{\gamma} = 0, \frac{1}{s}$.

Regarding the control one may alter, aside from the control-gearing ratio k discussed for the aileron moment l_φ, the measuring ratio: $m = 0.1, 0.2, 0.3$; for the control used here it depends on the angle formed by its measuring axis and the vertical axis of the glide bomb which is in these three cases approximately 6°, 11°, and 17°.

b. The Principal Case

Although the principal case which is characterized by a steady rectilinear flight path probably never actually occurs, it has its justified significance for an investigation like the present one, because it avoids as far as possible any peculiarity in any respect. Thus one does not take a certain, actually occurring phugoid motion as basis for it; one starts, on the contrary, from the conventional concept that this motion has, inversely, to be regarded as deviation from a steady rectilinear flight path and has, accordingly, only the significance of a special case.

The principal case, selected according to this point of view, may now be treated numerically, using the numerical data of the previous paragraph. One obtains a good survey always by plotting first the stable domain for the representation of which, according to p. 21, the lines $e = 0$ and $D_4 = 0$ are drawn into a $l_\varphi n_\beta$-plane. $e = 0$ is a hyperbola degenerated into a pair of straight lines and can easily be plotted into the $l_\varphi n_\beta$-plane. In contrast, $D_4 = (ab - c)(cd - be) - (ad - e)^2 = 0$ is of a higher degree in l_φ and n_β, and requires, for the numerical evaluation, some deliberation in order to obtain with a minimum of calculation expenditure sufficiently accurate results. The numerical treatment of such problems mostly is discussed rather cursorily; however, the attainment of final results is still a long way off, and thus a few useful remarks concerning the calculation will be inserted here.

First, it is always useful to clarify the desired accuracy in order to determine accordingly the number of digits required for calculation. Here the result ought to show three digits, the last of which may be uncertain; since, however, in the calculation process the first digit of many an important number is lost in the forming of differences, four digits are necessary to start with. Thus, the rounding up or off did extend not only to whole but also to half units of the last digit carried which was then denoted by Burrau's point 11 . This simple means permits, by the way, for the same number of digits an effortless doubling of the accuracy of a calculation if the necessity arises. Any carrying of further digits would, after all, be wasted effort if they finally are not expressed in the graphical representation and do not affect the result in any other way. By making suitable proofs during the lengthy calculation process, the always occurring unavoidable errors may be prevented from doing more extensive damage.

Figure 6 shows the thus obtained result, the stability domain of the principal case. The continued necessary determination of the stability at an arbitrary point of the represented $l_\varphi n_\beta$-domain outside of the negative (shaded) region of e and D_4 is quite simple: the coefficients a to d there directly turn out positive, just as the Hurwitz determinant $D_2 = ab - c$ which was already formed in calculating D_4.

Consideration of the principal case as well as of later cases shows quite uniformly that the moment coefficients l_φ for the aileron moment and n_β for the yawing moment due to sideslip (weathercock stability) are more or less equivalent with respect to lateral stability; in a representation at the same scale for l_φ and for n_β the approximate mirror image symmetry of the regions with respect to the bisector of the angle would become even clearer. A comparison of the results shows that this mirror image symmetry is more or less strongly influenced by various couplings and dampings about the longitudinal and vertical axis. Hence the equivalence of an immediately effective restoring moment for these two axes – which had been assumed previously in selecting the $l_\varphi n_\beta$-plane – proves correct; under similar circumstances it probably holds true for other methods of control as well.

The large stable domain of the principal case can be utilized practically only with a few restrictions. It will be useful – if only because of the uncertainty (discussed on p. 11) in actually maintaining a certain value of the weathercock stability – to select only operating points sufficiently far distant from the l_φ-axis, even if theoretically stability prevails up to their immediate proximity. For the aileron moment, conditions are not so sensitive; however, one will also select values at least ample enough that (in spite of the always possible inaccuracies of construction) no excessively erroneous positions remain permanently in existence. An upper limit for l_φ and n_β will be given mostly by the frequencies which the glide bomb is not to exceed with reference to its longitudinal and vertical axis.

A possible range chosen according to these deliberations might for instance lie at $l_\varphi = 1$ and $n_\beta = 0.3$; the stability for this case will now be investigated more closely. In order to be able to judge how it must be shifted if its behavior concerning the initial disturbance causing the oscillation is to be altered, and in order to gain a more exact insight into the distribution of frequencies and dampings in the stability domain, the latter are additionally determined along two sections at $l_\varphi = 1$ and $n_\beta = 0.3$.

First the real root of the frequency equation describing the yawing motion is determined for the entire $l_\varphi n_\beta$-region represented. As an approximation the value given on p. 23

$$\lambda = -\frac{c_a}{mT_F} = -\frac{0.35}{0.2 \times 4.48} = -0.39, \frac{1}{s}$$

was used, and the further correction was made according to Newton with the aid of Horner's scheme.

Figure 7 shows the result. As was to be expected according to previous deliberations, major deviations of the root λ from the above approximate value, otherwise accurate within 1 to 2 percent, occur in the proximity of the l_φ- and n_β-axis. This approximate value signifies that according to $\frac{\ln 2}{-\lambda} = 1.8s$ an error in course has been damped to half amplitude.

If one can make sure, in stability investigations of this or a similar kind, that one stays within the part of the $l_\varphi n_\beta$-plane only where this good agreement exists, it is very profitable beforehand to lower the degree of the frequency equation by one with the aid of this root. As a result, one need calculate only with much simpler Hurwitz determinants and the evaluation of the frequency equation is facilitated. Unfortunately, this method is not possible for the present investigation, because the stability domains are, for the sake of a complete survey, plotted to the l_φ- and n_β-axis.

If the root of the yawing motion has been determined with sufficient accuracy for an arbitrary point of the $l_\varphi n_\beta$-plane and is inserted for the last time as a proof into Horner's scheme, there result at the same time the coefficients a' to d' of the remaining frequency equation of the fourth degree

$$\lambda^4 + a'\lambda^3 + b'\lambda^2 + c'\lambda + d' = 0$$

Its roots describe the oscillations of the glide bomb about the longitudinal and vertical axis which in the proximity of the l_φ- and n_β-axis always become aperiodic; this equation now has to be solved numerically. The iteration method of v. Kármán-Trefftz [4] has the disadvantage that it converges poorly for roots of equal order of magnitude and for that reason is not always usable. In order to attain the aim quickly also for this case, an approximation method is given in the appendix which permits a rapid and clear solution. Figures 8 to 11 were calculated according to this method – chiefly according to the "pi – method" because it agrees better with the approximate values (pp. 23 and 24) used generally.

Figure 8 shows the section along $l_\varphi = 1$ through the stability domain of the principal case. No pecularities of any kind occur. The rolling oscillation remains almost unchanged; the yawing oscillation becomes slower and slower for smaller n_β-values and its two roots finally become real so that it is no longer noteworthy in this boundary region for vanishing yawing moment due to sideslip at the limit of static stability. The damping of the yawing oscillation meanwhile increases more and more rapidly and would exceed the value 1 in the aperiodic limiting case.

The approximate values for Π and π agree well, those for Σ and σ only just tolerably, with the exact ones, as can be seen from figure 9.

The section along $n_\beta = 0.3$ through the stability domain of the principal case shows a somewhat different picture insofar as one approaches here, with diminishing aileron moment, the limit of dynamic stability, as can also be seen from figure 6. Thus one damping vanishes there, in this case that of the rolling oscillation. No further points were calculated below the value $l_\varphi = 0.3$, where this occurs, since they have no practical significance. The fact that the damping, which finally disappears at the stability limit, starts decreasing more or less strongly before that, offers another clue for the selection of the operating point where the dampings – not too good as it is – are to be utilized as efficiently as possible.

The agreement of the approximations with the exact values of Π, Σ, π, and σ can be seen from figure 11, which again shows the good serviceability of the approximate values of Π and π determining the frequencies, and the not so good serviceability of those of Σ and σ decisive for the dampings.

What was said above concerning the root λ of the yawing motion and the variation of the frequencies and dampings applies, with the required changes, to any other of the cases discussed below that result from the principal case, and need not be repeated.

c. Influences of the Glide Bomb Values

Starting from the principal case one now varies in turn one quantity, as described on p. 19, and investigates its influence on the stability domain. All values used in this problem were thus included in arbitrary sequence, no matter whether their variation can be achieved easily or not at all; reasons for this will be given later.

First, the influence on the stability domain is investigated for steady rectilinear flight performed with different c_a and accordingly varied velocity, the other ratios remaining unchanged. As always in uncontrolled lateral motion here also a distinct c_a dependence becomes apparent: for smaller c_a-values the stability domain increases.

For the important determination of the variation of the stability domain during a phugoid oscillation one made the assumption that the release velocity of the glide bomb is only 60 percent of its equilibrium velocity; thus it was based on a very pronounced path oscillation which shows at the four locations denoted in figure 13 the following roughly calculated value:

TABLE 1.— VALUES DURING THE PHUGOID

Location	$v \frac{m}{s}$	γ	$\dot{\gamma} \frac{1}{s}$	$(n_x - T_F\dot{\gamma})$
1 wave crest	75	−0.146	−0.0628	0.329
2 descending flight	125	−.589	0	−.140
3 wave trough	156	−.146	.0386	−.279
4 ascending flight	125	.126	0	−.140
steady rectilinear flight	125	−0.146	0	−0.140

Flight in the wave trough is found to be most unfavorable for later lateral stability. The considerable difference between the stability domains of the wave crest and the wave trough stems solely from the additional gyroscopic moment due to the rotation of the path of the longitudinal motion and shows clearly that the taking into consideration of the gyroscopic term which was motivated on p. 6 is justified. Since it appears only together with the rolling-yawing moment n_x in the combination $(n_x - T_F\dot{\gamma})$, — as both vary with ω_x — the two stability domains for the wave crest and the wave trough may be interpreted also as if in the steady rectilinear flight of the principal case the n_x were one time equal to +0.329 and the other time equal to −0.279. Thus the influence of the rolling-yawing moment on the stability domain is simultaneously clarified and need not be investigated separately.

Compared to the locations 1 and 3 where the path rotation of the longitudinal motion is most pronounced, the approximately linear parts of the phugoid motion at the locations 2 and 4 exert hardly any influence on the stability domain. The longitudinal inclination of the flight path γ as such, is, therefore, insignificant provided the c_a-value remains the same.

An actually performed flight of a glide bomb with the approximate values of the principal case is represented in figure 14. The aileron moment is about $l_\varphi = 1$, the weathercock stability $n_\beta = 0.5$; it is true, the reduction in velocity at the release is not as high as had been presupposed in figure 13 so that the phugoid motion appears less marked. The lateral curvatures of the path at about 30 to 40 and 50 to 60 seconds flight duration stem from arbitrary commands which are to be interpreted as disturbances for the lateral motion in the sense of p. 5.

Another value besides the control-gearing ratio k which is variable by the control is the measuring ratio m; for the type of control used here it depends on the tilt of its measuring axis in the glide bomb. Figure 15 shows that in the case $m = 0.1$ (too slight tilt) a large part of the useful stability range is lost because one is already too close to the limiting case $m = 0$ where a bank is no longer measured and the control method is therefore useless. Also, for $m = 0.3$ (excessive tilt) conditions are somewhat more unfavorable than for $m = 0.2$. Thus an optimum regarding the magnitude of the stability range lies between $m = 0.1$ and $m = 0.3$, probably in the proximity of the value used for the principal case $m = 0.2$. Since the time to damp half the amplitude of the yawing motion varies with m, as can be seen from pp. 28 and 29, it is from this point of view expedient to use a small m, thus a small tilt.

The influence of the rolling moment due to sideslip may be best understood on the basis of the conventional representation of the stability range for the uncontrolled lateral motion in the $l_\beta n_\beta$-plane as it can be found for instance in Mathias [12]. For this purpose the representation of the stability range for the principal case without control, thus for $l_\varphi = 0$, was incorporated in figure 16. If one considers in it, for instance, the point 0.3 on the n_β-axis, one is in the statically unstable range and, as is well known, it takes a definite minimum rolling moment due to sideslip l_β to attain stability. If one proceeds in the direction of increasing l_β-values, one reaches – it is true, for rolling moments due to sideslip so large that one would no longer consider them, for instance, for airplanes of ordinary design – again an unstable region, which this time, however, is the region of dynamic instability. If one stops at $l_\beta = 4$, one has reached approximately the rolling moment due to sideslip of the automatic pilot considered here, which is, therefore, much too large. In order to attain stability, the control must intervene with a minimum aileron moment l_φ, which would be read off for $n_\beta = 0.3$ from figure 16a. For $l_\beta = 2$ and the same n_β the glide bomb with the values taken here as a basis would be stable also without control, and only below $n_\beta = 0.15$ a control would become necessary. The case is slightly different for $l_\beta = 0$. Here the uncontrolled glide bomb is not stable for any positive n_β; however, a very small l_φ on the part of the control system is sufficient to eliminate this static instability; hence in figure 16a the different character of the unstable range for $l_\beta = 0$ and the two other l_β-values. It is shown with particular emphasis how many advantages a small rolling moment due to sideslip offers, which probably applies likewise to glide bombs of similar type.

The weight of the glide bomb may be increased for unchanged lift coefficient c_a and correspondingly increased flight velocity or inversely. The first case is represented under the assumption that the radii of inertia remain unchanged; it shows that by using this method surprisingly little of the stability range is lost.

As discussed on p. 26, the same over-all picture would hold if for unchanged weight the flight altitudes were instead of the original 1.0 kilometer (principal case) now 4.5 and 6.9 kilometers. This fact signifies only that the stability range remains fairly constant when large differences in altitude are passed through in flight.

Figure 18 represents the flight of a glide bomb of about 1000 kilograms weight to which corresponds, for an altitude of 2.0 kilometers, a relative aircraft mass density $\mu_s = 1300$. The initial lateral curvature of the path was not intentional but was probably due to release disturbances. For a prescribed size of the glide bomb, the high flight velocities of more than 200 meters per second occurring here can be avoided only by an increase of the lift coefficient c_a, as had been presupposed for figure 19.

Here an increase in weight, for unchanged radii of inertia, is obtained at the price of a much more extended enlargement of the unstable region, as had been the case for unchanged lift coefficient c_a. Comparison with figure 12 where only the variation of c_a was investigated, while the weight remained unchanged, shows a striking similarity in the unfavorable influence of high c_a-values on the stability region. Thus the assumption suggests itself that of the possible combinations of lift coefficient, flying weight, and velocity the first exerts the decisive influence on the size of the stability domain. In order to recognize a possibly existing connection, the table 2 was set up:

TABLE 2.- RATIO OF THE SIZE OF THE UNSTABLE REGION AND DIFFERENT STATES OF FLIGHT FOR RECTILINEAR PATH

Figure	Size of the unstable region	c_a	μ_s	Fr	T_F s
12	0.014	0.15	700	2340	2.93
	.117	.35	700	1000	4.48
	.270	.55	700	610	5.66
13 location 2	0.122	0.35	700	1000	4.48
13 location 4	.120	.35	700	1000	4.48
17	0.142	0.35	1000	1430	5.35
	.162	.35	1300	1850	6.11
19	0.259	0.497	1000	1000	6.40
	.397	.643	1300	1000	8.32

The rectilinear but not steady parts of the ascending and descending flights during the phugoid motion also were included with the cases suitable for this comparison. As measure of comparison for the unstable region we selected the size of its area in the represented $l_\varphi n_\beta$-region, expressed in $(l_\varphi n_\beta)$-units. The weight was expressed by the relative aircraft mass density μ_s and the flight velocity by the Froude number $Fr = v^2/bg$. Finally, the aerodynamic time unit $T_F = \mu_s \frac{s}{v}$ was included since it belongs in this example.

As even a most superficial consideration reveals, the magnitude of the unstable region is an approximately unequivocal function only of the lift coefficient c_a; this dependence is represented in figure 20.

Without generalizing too hastily it is, therefore, shown with satisfactory regularity that with respect to the magnitude of the unstable region, small c_a-values are favorable, large ones unfavorable. Accordingly, it will be best, if a minimum unstable region is desired, to fly with a small c_a-value, mostly given by the maximum admissible flight velocity. If the longitudinal motion of the glide bomb is controlled arbitrarily or automatically, the unfavorable influence of large angles of attack maintained for a longer time must be taken into consideration in a given case.

At this point it should be mentioned explicitly that in case of a change of c_a the two related coupling moments l_z and n_x were also changed throughout, because the same holds true actually. The influence of the rolling moment due to yawing l_z as such is insignificant, as figure 25 will show, and the rolling moment due to yaw n_x also is only of secondary importance compared to c_a. It must be borne in mind that in figure 13 an n_x that should appertain to $c_a = 0.70$ corresponds to the slightly enlarged unstable region of the wave trough. Hence it results that it is really the lift coefficient c_a, and not one of the coupling moments l_z and n_x dependent on it, which exerts the decisive influence on the size of the unstable region.

Figure 21a shows the expected favorable influence of a large damping-in-roll. For $l_x = 4$, where the moment coefficients of the damping-in-roll and damping-in-yaw are equal, the mirror image symmetry with respect to the bisector of the angle mentioned on p. 28 is strikingly good; this stands out clearly in figure 21b where l_φ and n_β are plotted to the same scale. If the damping-in-roll is improved, the unstable region is reduced in such a manner that it decreases more in l_φ- than in n_β-direction (again under the assumption of equal scale!)

The variation of the damping-in-yaw now also has the expected influence, as will be shown by the following case.

Here, too, the self-evident favorable effect of large damping-in-yaw is obvious; the unstable region is influenced mainly in its extension parallel to the n_β-axis.

One is not always able to make the damping-in-yaw – and even less the damping-in-roll! – so large as to obtain a decisive effect on lateral stability. The investigation of these and also of other cases where an influencing of the aerodynamic properties of the glide bomb is, in practice, hardly possible has no other purpose than to determine which one of the two limiting cases of numerical data known only inaccurately must be substituted into the calculation to stay on the safe side.

The lateral force which, among other effects, also has a damping effect, exerts a surprisingly small influence, the reason being that it is by one order of magnitude smaller than the moment coefficients of the damping-in-roll and damping-in-yaw.

For negative n_ξ / l_ξ, that is, when the restoring force about the vertical axis resulting from the weathercock stability is reduced because of the aileron deflections simultaneously produced by the control, the coefficient e may become negative in spite of positive (but too small) weathercock stability. This case discussed on pp. 17 and 18 is practically the only one where static lateral instability also is possible.

For positive n_ξ / l_ξ, on the other hand, the region of dynamic instability is enlarged so that – the aileron yawing moment being not definitely known – it is best to take both influences into consideration.

The rolling moment due to yawing has practically no influence at all on the magnitude of the stability domain, as is to be expected (if for no other reason) because of the very full wing shape of the glide bomb. It is related to ω_z, exactly like the gyroscopic moment that would arise for a path rotation of the longitudinal motion and, accordingly, would be just as insignificant. Thus it is shown also from this point of view that the neglect of the gyroscopic term in the equation of moments for the longitudinal axis was fully justified.

d. Conclusion

In surveying the figures which represent the stability domains as functions of the various glide bomb values, one finds that directional stability is with certainty attainable for every case, by appropriate adjustment of the control in cooperation with the ailerons (m, l_φ) and a corresponding yawing moment due to sideslip (n_β). No other definite aerodynamic properties of the system need be assumed to make the control method treated here serviceable although some are more or less unfavorable. Of course, one will, where it is easily possible, select the more advantageous design; the following table may be used as a directive.

TABLE 3.– INFLUENCES ON THE LATERAL STABILITY

Parameter	Figure	Favorable	Unfavorable
Lift coefficient c_a	12	Small	Large
Phugoid	13	Wave crest	Wave trough
Rolling–yawing moment n_x	13	Large (> 0)	Small (< 0)
Measuring ratio m (a function of gyroscope–tilt angle)	15	About 0.2	Other values
Rolling moment due to sideslip l_β	16	Small	Large
Weight increase (μ_s)	17, 19	Small c_a	Large c_a
Altitude H	17	Small	Large
Damping–in–roll l_x	21	Large	Small
Damping–in–yaw n_z	22	Large	Small
Lateral–force increase c_q	23	Large	Small
Aileron yawing moment n_ξ	24	About 0	Other values
Rolling moment due to yawing l_z	25	Practically without influence	

The table is compiled without consideration of the question whether or not an alteration is actually feasible for the individual case; at the same time, the table should be used to select, in case of unreliably known values, always the most unfavorable values for safety reasons. Most significant with regard to the minimum possible unstable region for lateral motion of the investigated controlled glide bomb are small rolling moment due to sideslip l_β and small lift coefficient c_a.

As a conclusion it should be mentioned again, in regard to what was said on p. 5, that the object of this investigation was not to present, for a certain individual case, a calculation as accurate and complete as possible, but rather to give a survey so comprehensive that its results make a more general insight into the problems of this automatically controlled glide bomb possible.

VII. SUMMARY

The investigation of the lateral stability of an automatically controlled glide bomb led also to the attempt of clarifying the influence of a phugoid oscillation – or of any general longitudinal oscillation – on the lateral stability of a glide bomb. Under the assumption that its period of oscillation considerably exceeds the rolling and yawing oscillation and that c_a is, at least in sections, practically constant, the result of this test is quite simple. It becomes clear that the influence of the phugoid oscillation may be replaced by suitable variation of the rolling-yawing moment on a rectilinear flight path instead of the phugoid oscillation. If the flying weight of the glide bomb of unchanged dimensions is increased, an increase of the flight velocity will be more favorable than an increase of the lift coefficient. The arrangement of the control permits lateral stability to be achieved in every case; a minimum rolling moment due to sideslip proves of great help.

VIII. APPENDIX

AN APPROXIMATION METHOD FOR SOLUTION OF ALGEGRAIC EQUATIONS OF THE FOURTH DEGREE

One imagines the four roots of a prescribed algebraic equation of the fourth degree

$$\lambda^4 + a'\lambda^3 + b'\lambda^2 + c'\lambda + d' = 0$$

grouped into two pairs; let the root products and sums for each of these pairs be denoted by Π, Σ and π, σ, respectively. If two conjugate-complex roots exist, their combination is self-evident and real roots are treated as pairs, grouped arbitrarily by twos.

The following ratios are known to exist between the coefficients a' to d' and the root products and sums:

$$a' = -\Sigma - \sigma$$

$$b' = \Pi + \Sigma\sigma + \pi$$

$$c' = -\Pi\sigma - \pi\Sigma$$

$$d' = \Pi\pi$$

They readily permit the following method of solution for the four new real unknowns Π, Σ, π, and σ to be read off:

For instance, let an approximate value of Π (or π) be known; then there results first, from the fourth equation, π (and Π, respectively) and then, as can be seen easily, from the combined first and third equation, the σ and Σ. If the Π (or π) used initially had been correct, the result (in case of substitution of all these successively found values into the right side of the second equation) would be exactly b'; but since an approximation had been used, the proof $\Pi + \Sigma\sigma + \pi - b' \neq 0$ will be omitted. Thus the result of the proof is in a very simple manner dependent on Π (or π); this value can now be easily determined according to the regula falsi (bracketing) with arbitrary accuracy in such a way that the proof gives zero; then one has found the correct value of Π (or π). Thus the course of calculation appears as follows, starting from an approximate value for Π:

$$\Pi;\ \pi = \frac{d'}{\Pi};\ \sigma = -\frac{c' - a'\pi}{\Pi - \pi};\ \Sigma = -(a' + \sigma);\ \Pi + \Sigma\sigma + \pi - b$$

Every further line is used to obtain a more and more exact value of Π until the desired accuracy is reached. If one finally inserts the correct value for Π, one obtains, according to this method, not only the remaining values π, σ, and Σ, but has in addition for all of them a common proof. If one starts with π instead of Π, the calculation takes, after determination of $\Pi = d'/\pi$, exactly the same course. For Π and π being approximately equal, a more accurate determination of the second value of each line is sufficient to have, in forming the difference $\Pi - \pi$, enough digits left to calculate σ.

It is true that this method will fail in case of $\Pi = \pi$; however, the approximation method may be used here also, provided one starts, in a slightly varied manner, with Σ (or σ) instead of Π (or π). With the aid of the four ratios used initially the following second type of solution is just as clear:

After selection of Σ (or σ), the σ (or Σ, respectively) immediately results from the first equation, and then, from the second and third together, the π and Π; the proof $\Pi\pi - d' \neq 0$ now serves exactly as in the method described before for improvement of the initial value Σ (or σ). The separate lines of the calculation then read, if one starts, for instance, with Σ:

$$\Sigma; \; \sigma = -(a' + \Sigma); \; \pi = \frac{c' + \sigma(b' - \Sigma\sigma)}{-(\Sigma - \sigma)}; \; \Pi = b' - \Sigma\sigma - \pi; \; \Pi\pi - d'$$

This "Sigma-method" also has a weak point, in the case of $\Sigma = \sigma$. However, in that case it is very well supplemented by the "Pi-method", except for the (practically highly improbable) possibility of simultaneous equivalence $\Pi = \pi$ and $\Sigma = \sigma$. Anyway, this fact would be shown by definite ratios between the coefficients a' to d', and the solution is then always possible in some other manner.

With the values Π, Σ, π, and σ the problem is practically solved, since the four desired roots now can be easily determined from the two quadratic equations

$$\lambda^2 - \Sigma\lambda + \Pi = 0 \quad \text{and} \quad \lambda^2 - \sigma\lambda + \pi = 0$$

Thus only the character of the roots – whether real or complex – emerges; this knowledge is, therefore, not even required for performing this approximation method.

Translated by Mary L. Mahler
National Advisory Committee
for Aeronautics

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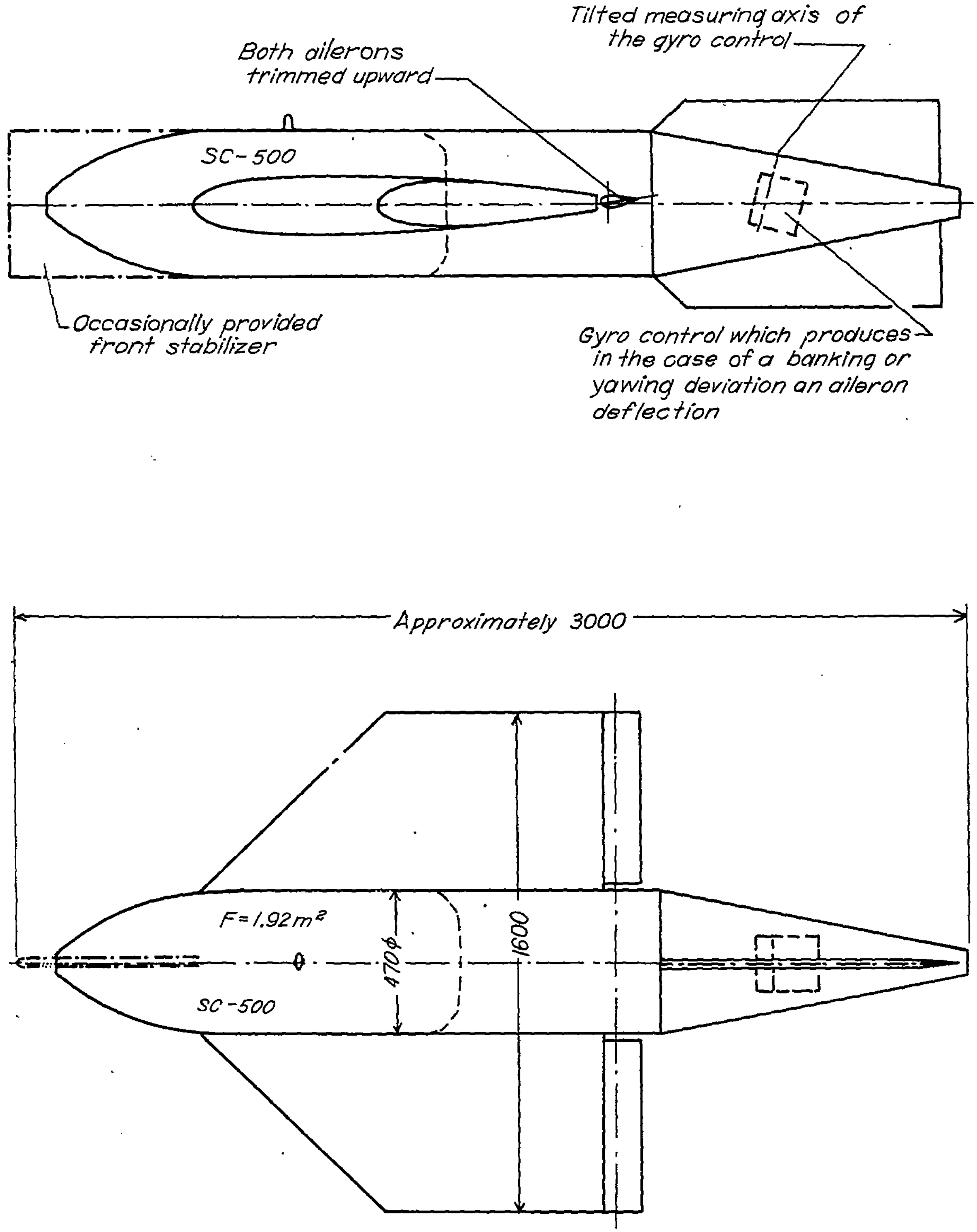

Figure 1.- The subject glide bomb (scale 1:20 dimensions in cm.)

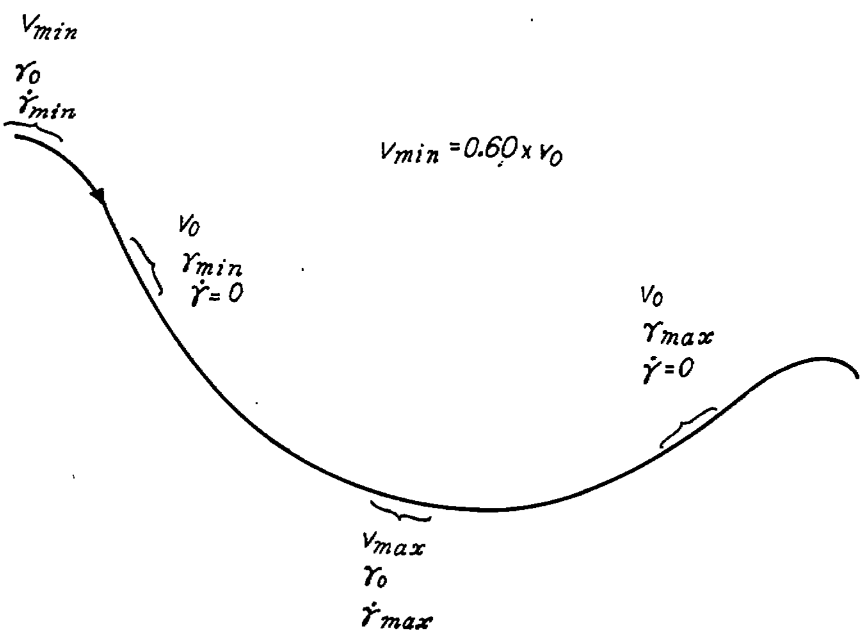

Figure 2.- The path oscillation (phugoid). (Not true to scale.)

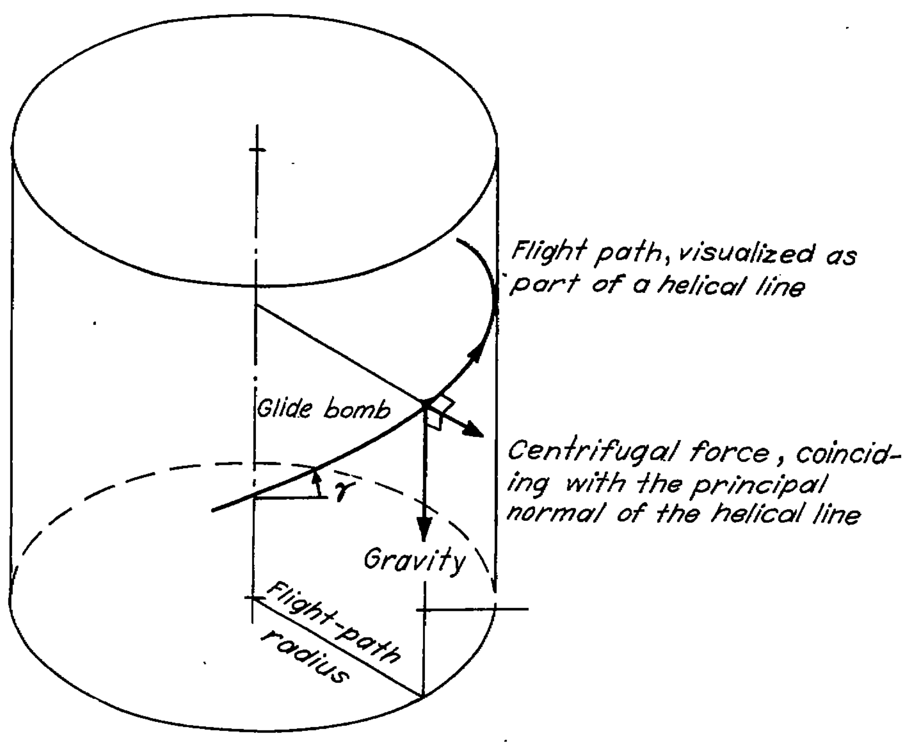

Figure 3.- Concerning the lateral motion of the glide bomb.

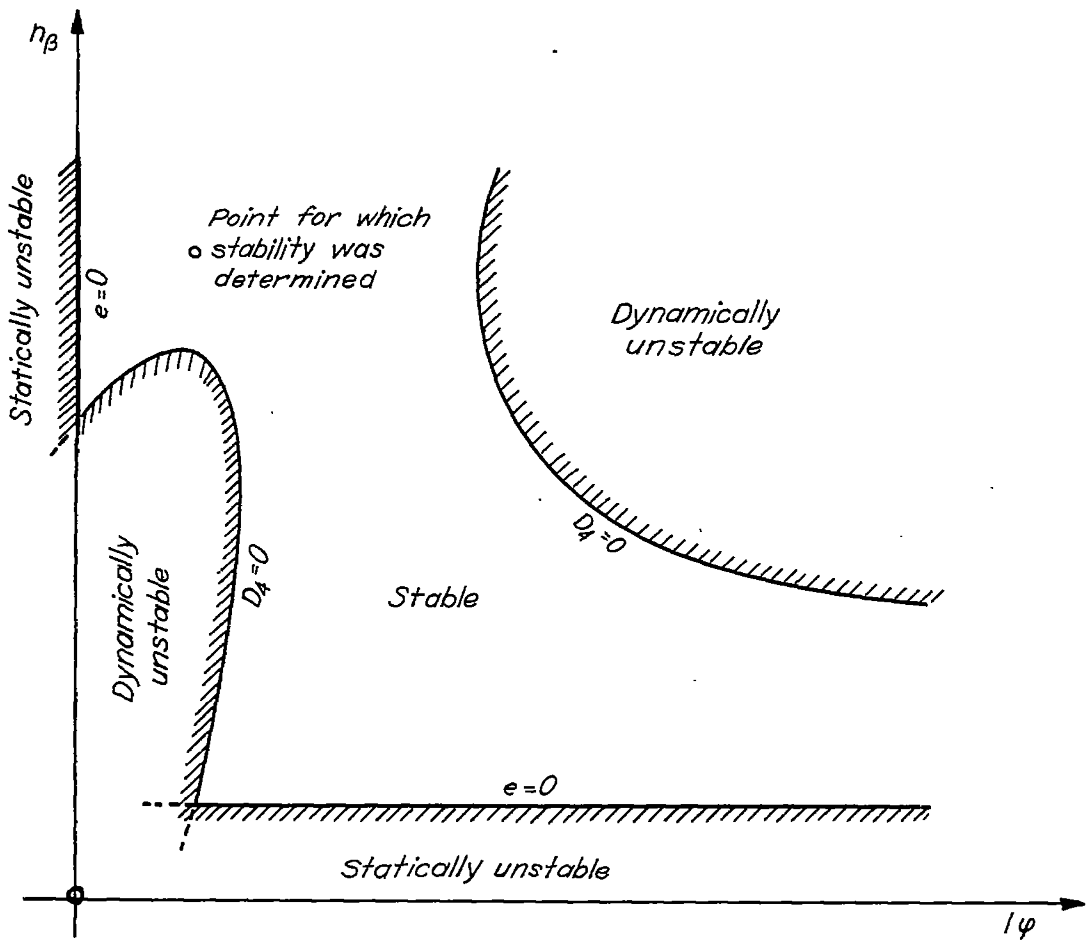

Figure 4.- The stability domain.

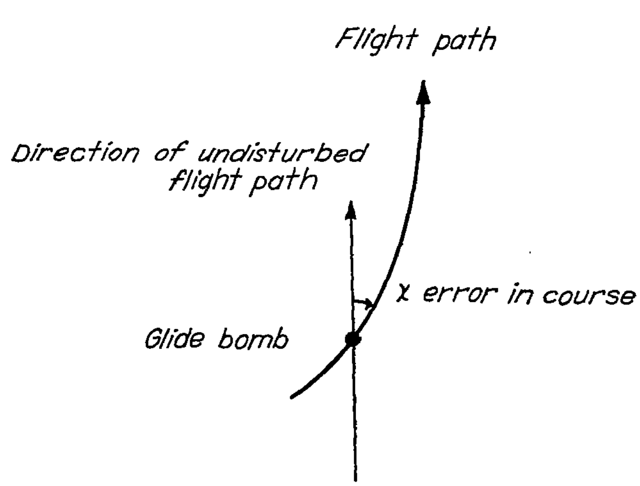

Figure 5.- The yawing motion.

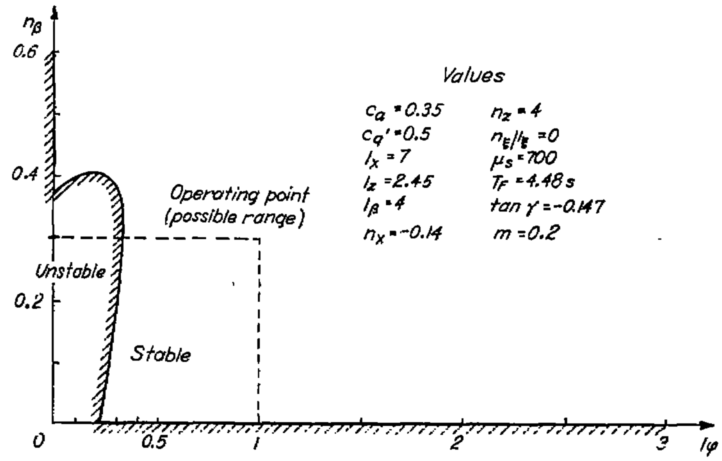

Figure 6.- The stability domain of the principal case, that is, for average values in steady rectilinear flight.

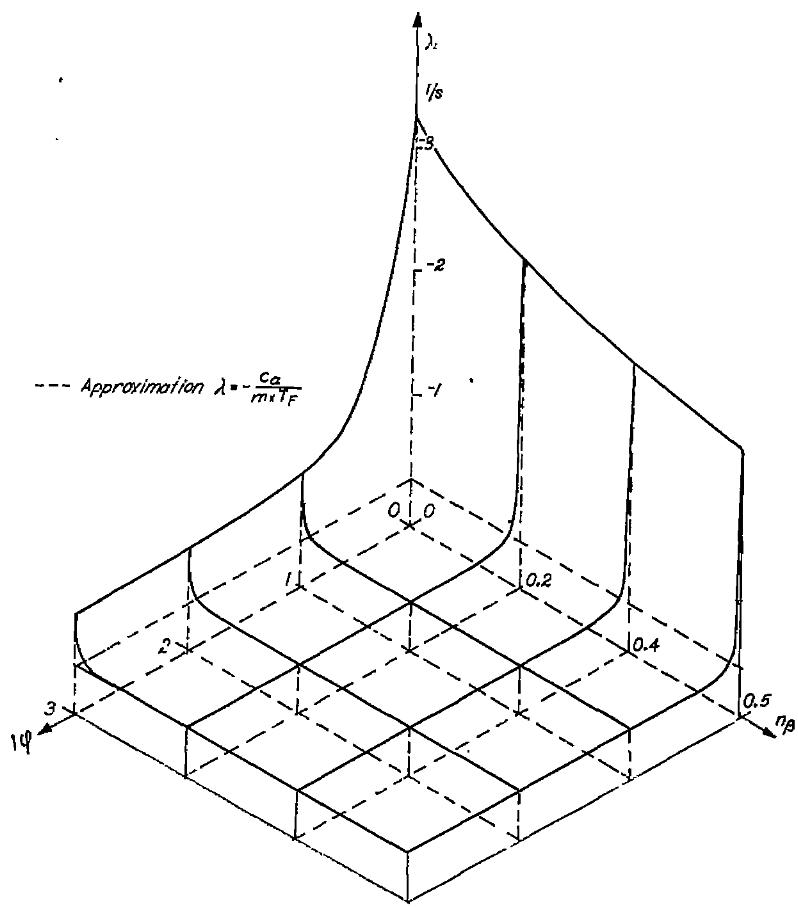

Figure 7.- The root λ of the yawing motion.

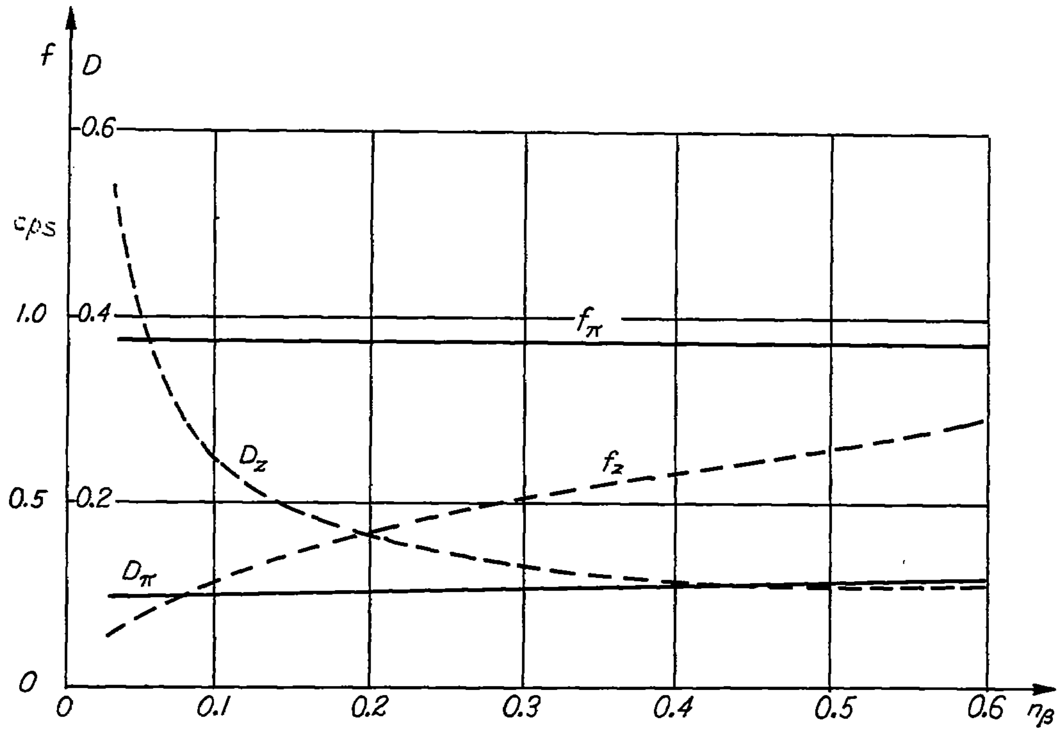

Figure 8.- Frequencies and dampings ($1_\varphi = 1$).

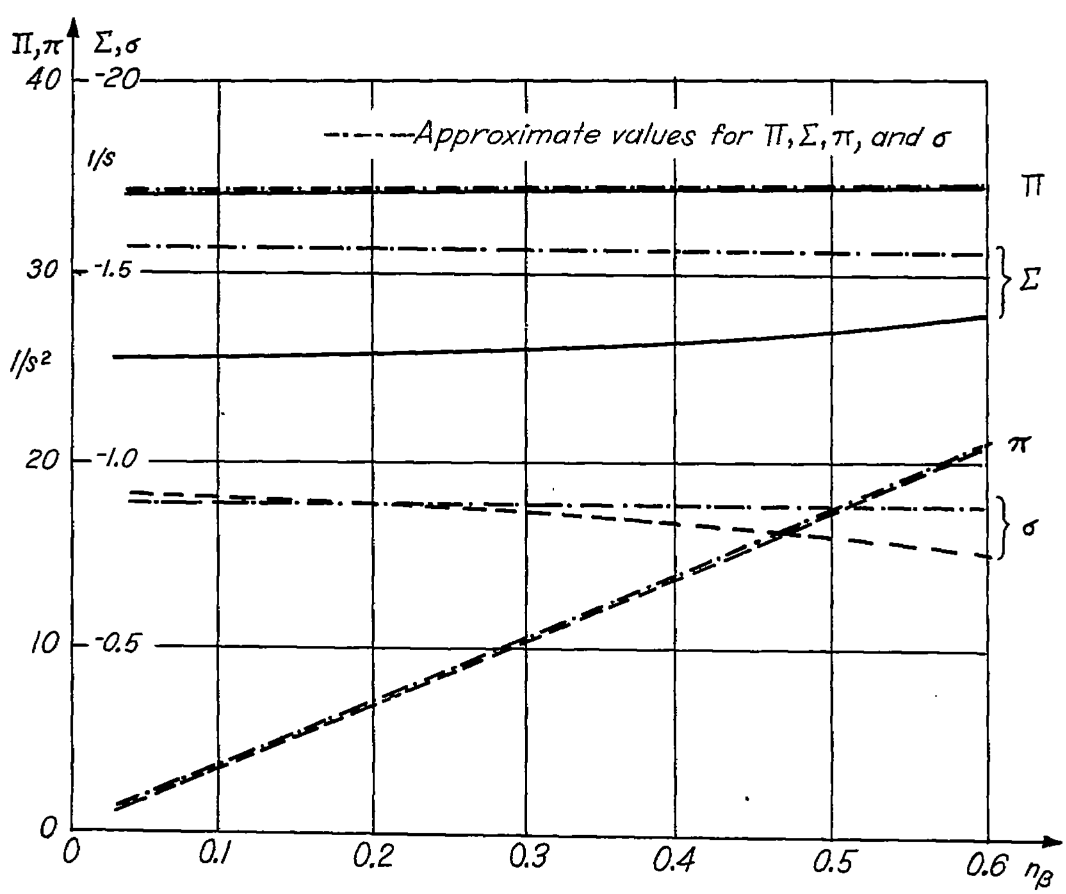

Figure 9.- Π, Σ, π, σ and their approximations ($1_\varphi = 1$).

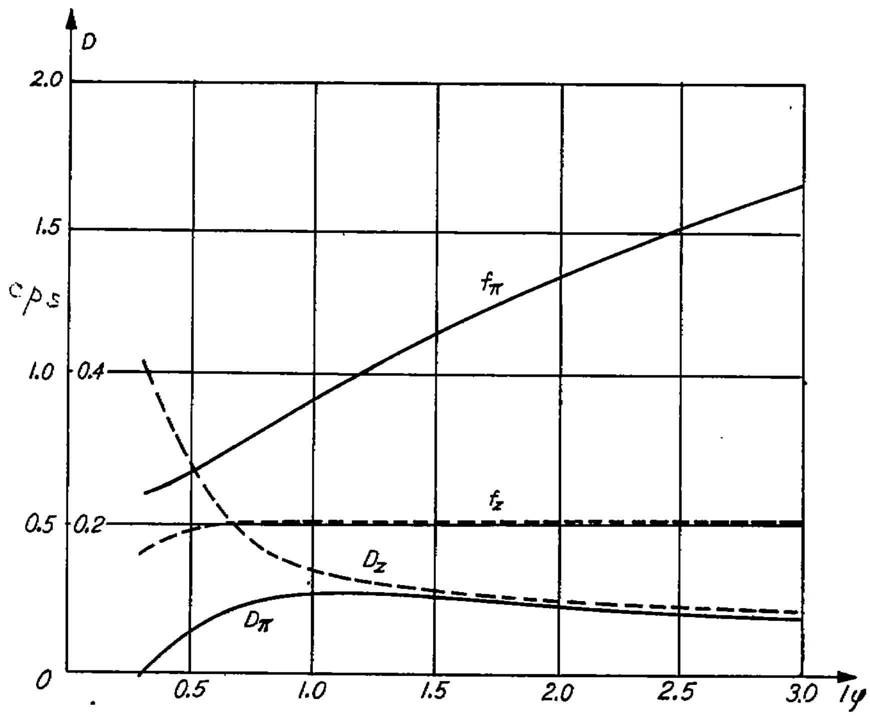

Figure 10.- Frequencies and dampings (n_β = 0.3).

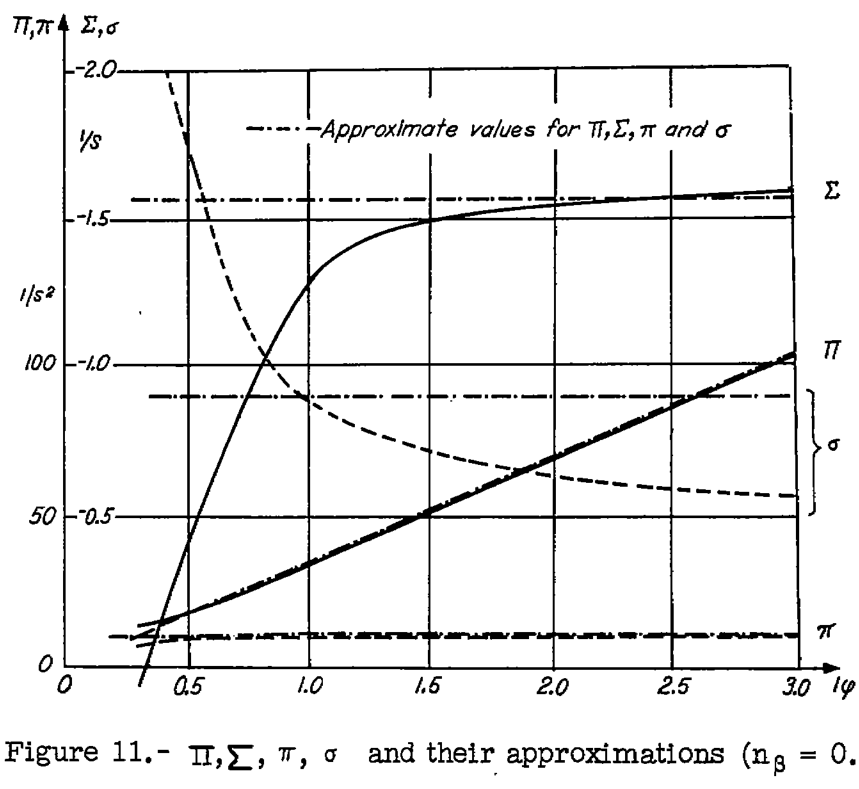

Figure 11.- Π, Σ, π, σ and their approximations (n_β = 0.3).

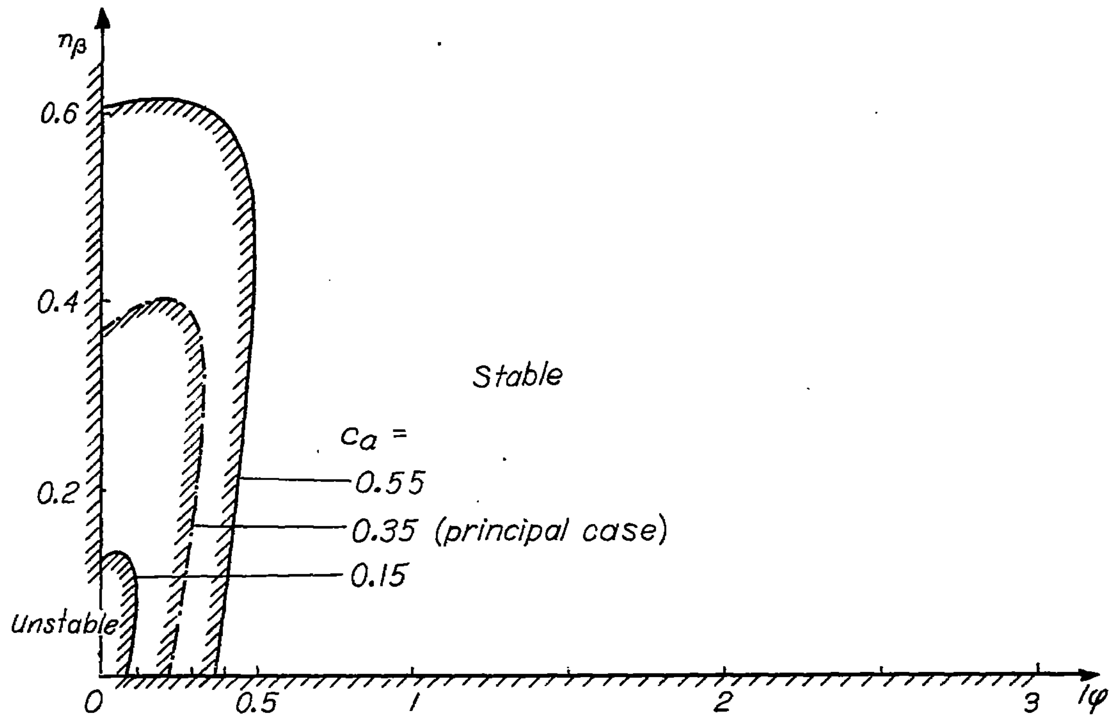

Figure 12.- Influence of the lift coefficient c_a on the stability domain.

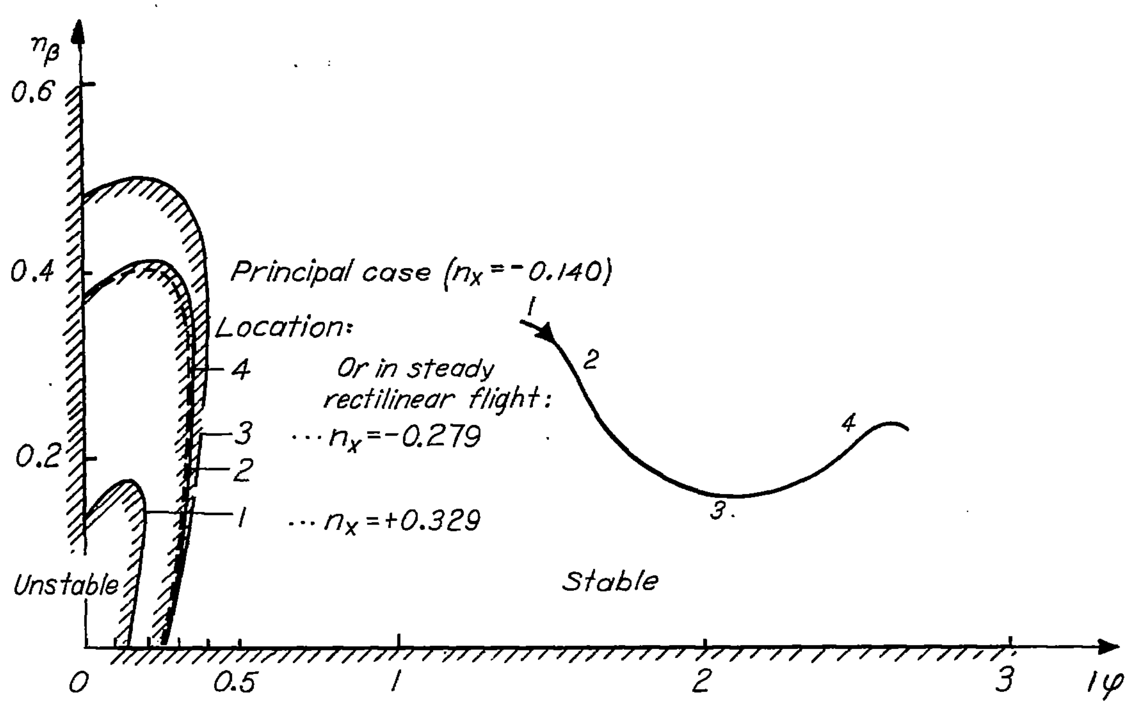

Figure 13.- Influence of the phugoid oscillation on the stability domain.

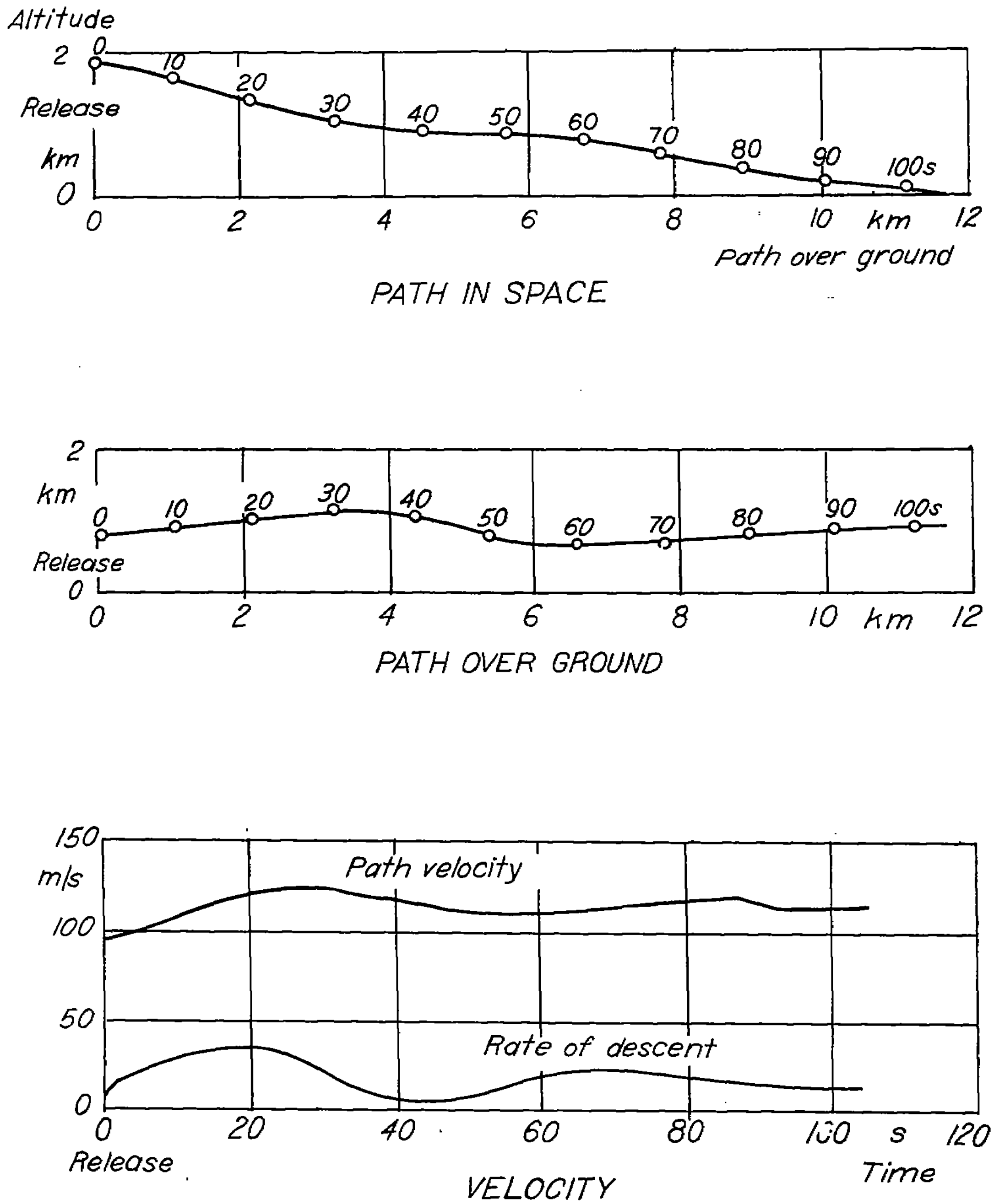

Figure 14.- Path and velocity of a glide bomb with $\mu_s = 700$ (principal case).

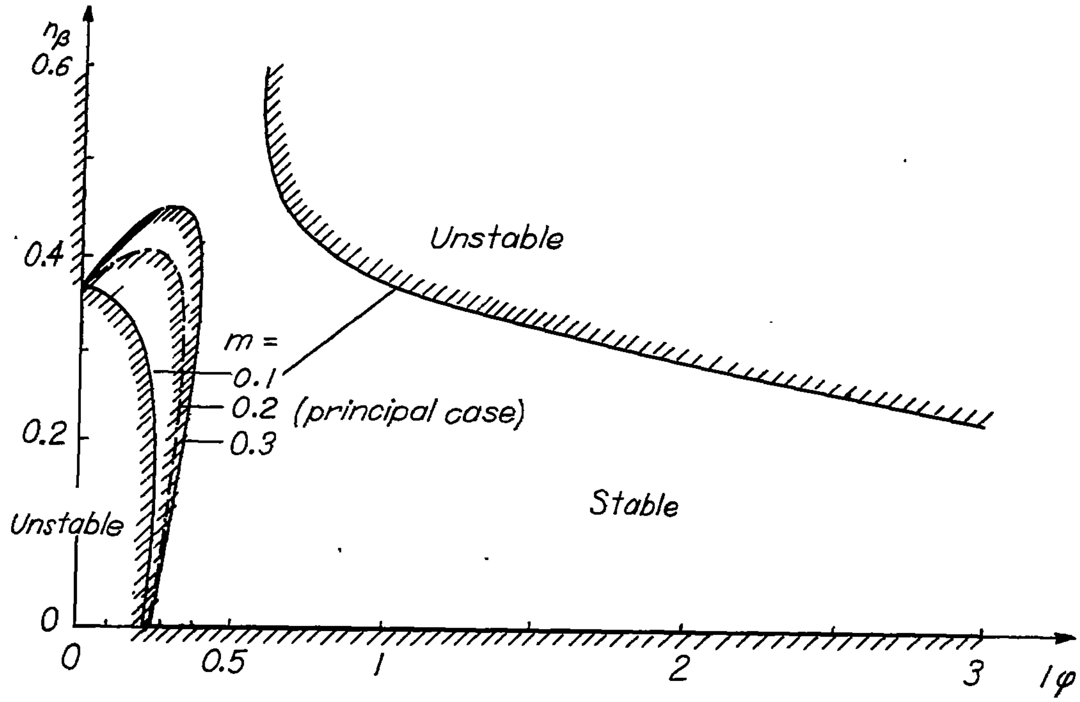

Figure 15.- Influence of the measuring ratio m on the stability domain.

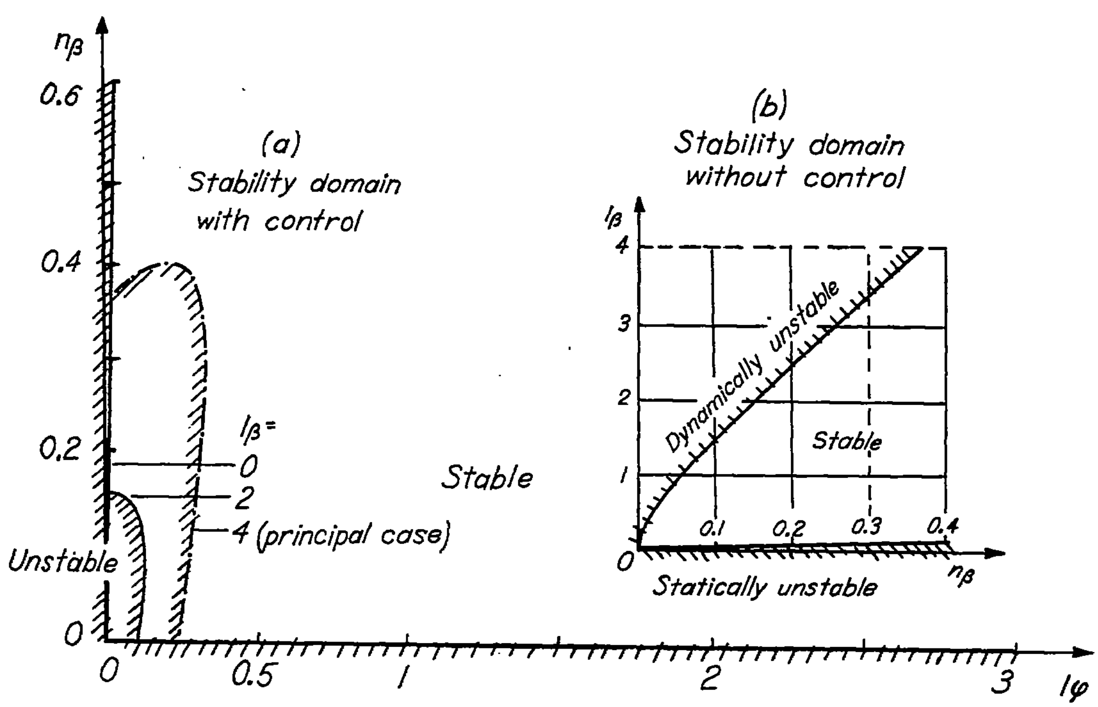

Figure 16(a) and (b).- Influence of the rolling moment due to sideslip l_β on the stability domain.

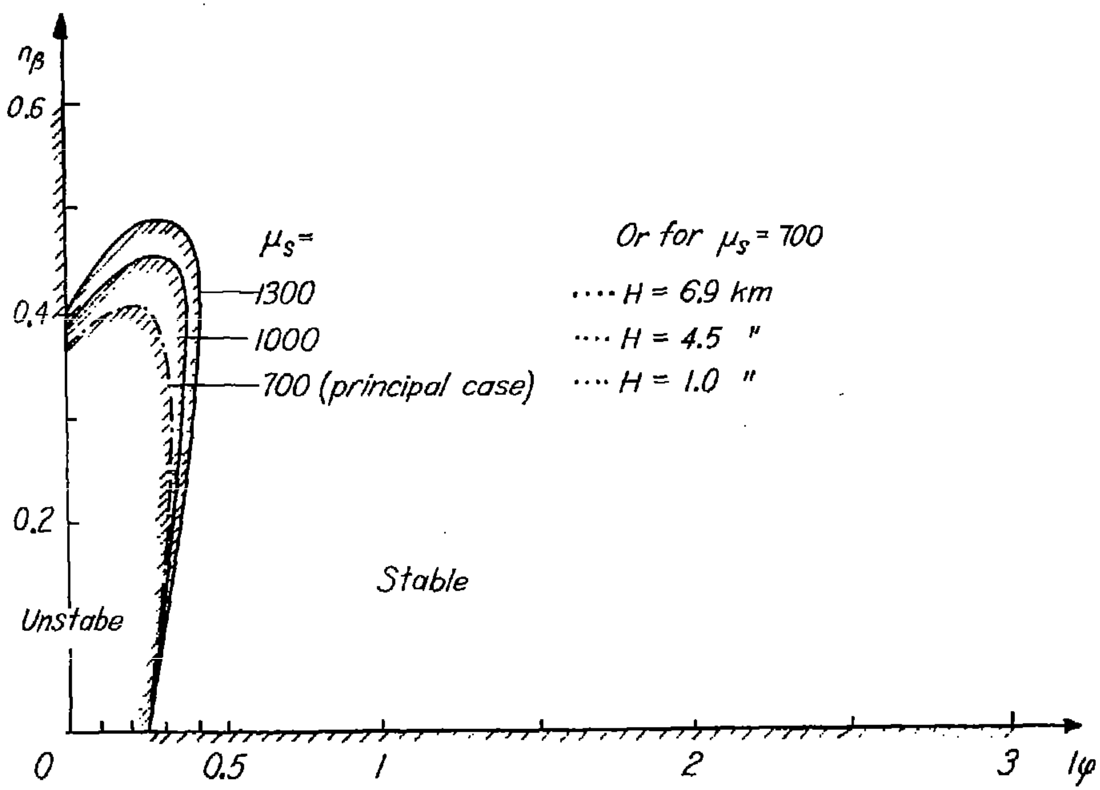

Figure 17.- Influence of the weight (c_a constant) on the stability domain.

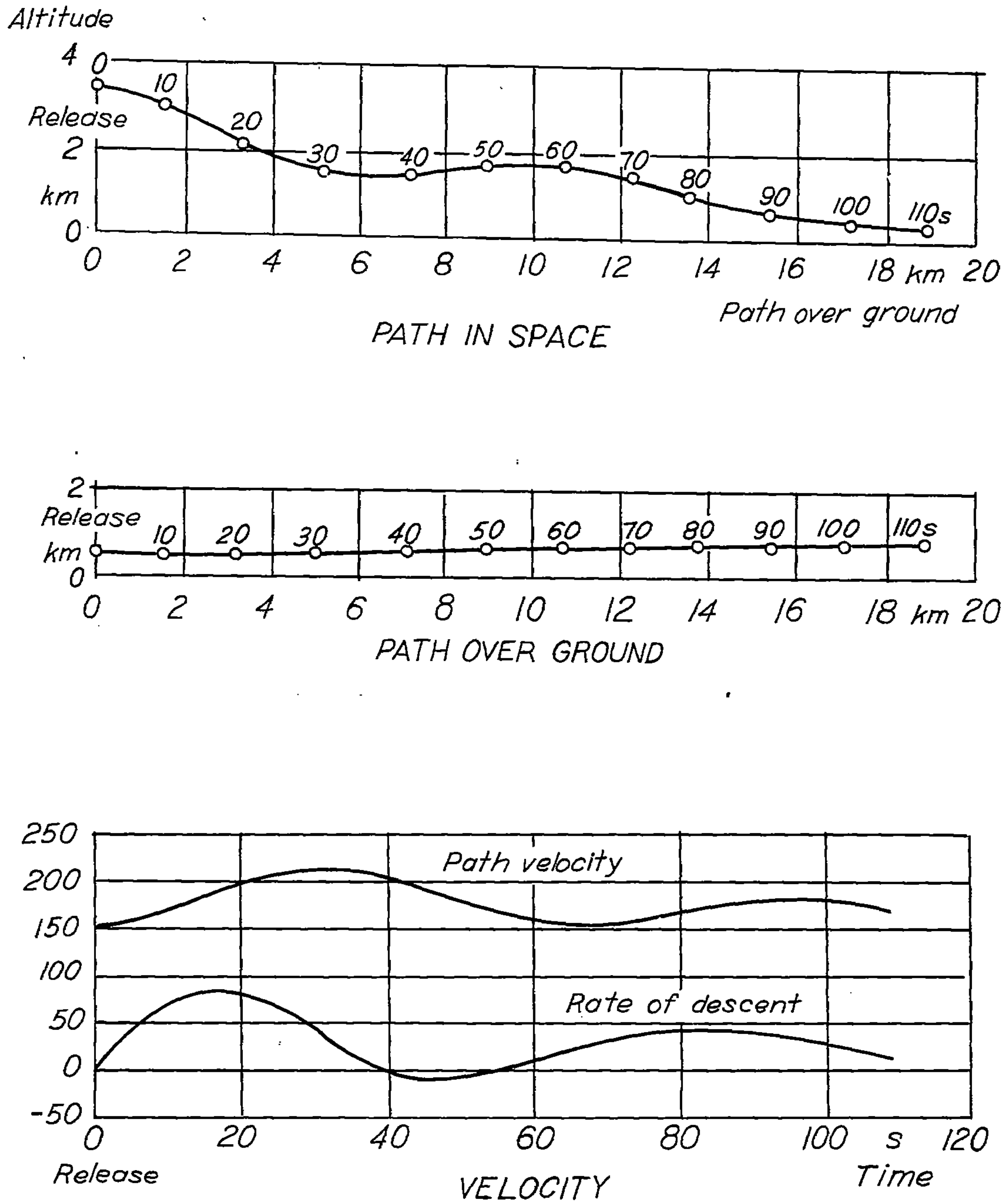

Figure 18.- Path and velocity of a glide bomb with μ_s = 1300.

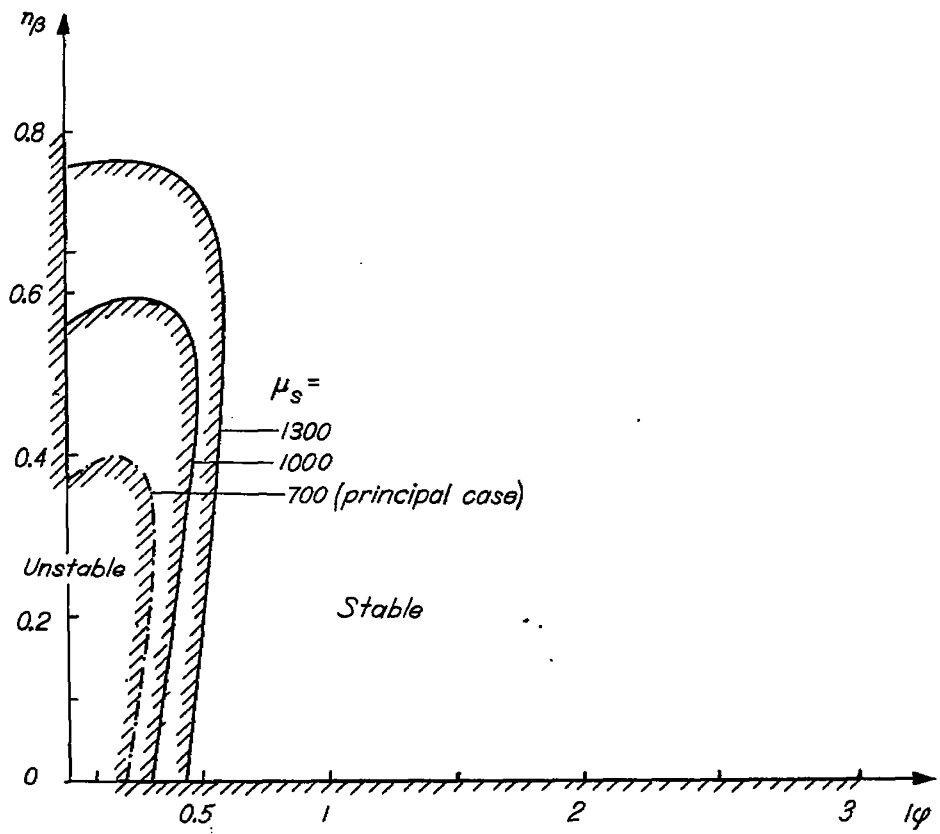

Figure 19.- Influence of the weight (v constant) on the stability domain.

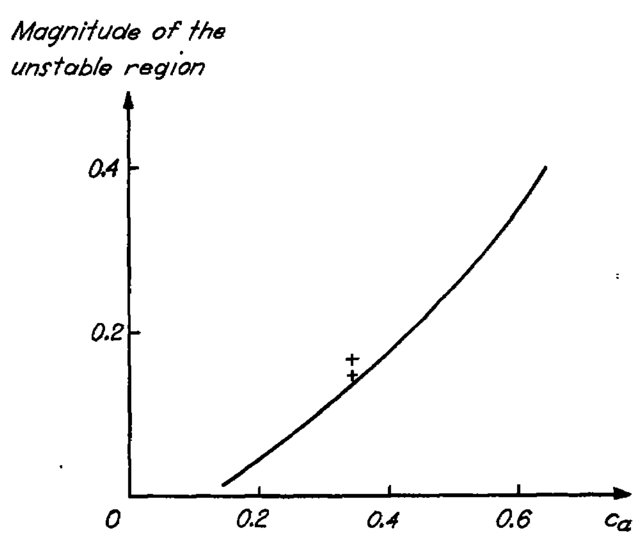

Figure 20.- Magnitude of the unstable region as a function of the lift coefficient c_a.

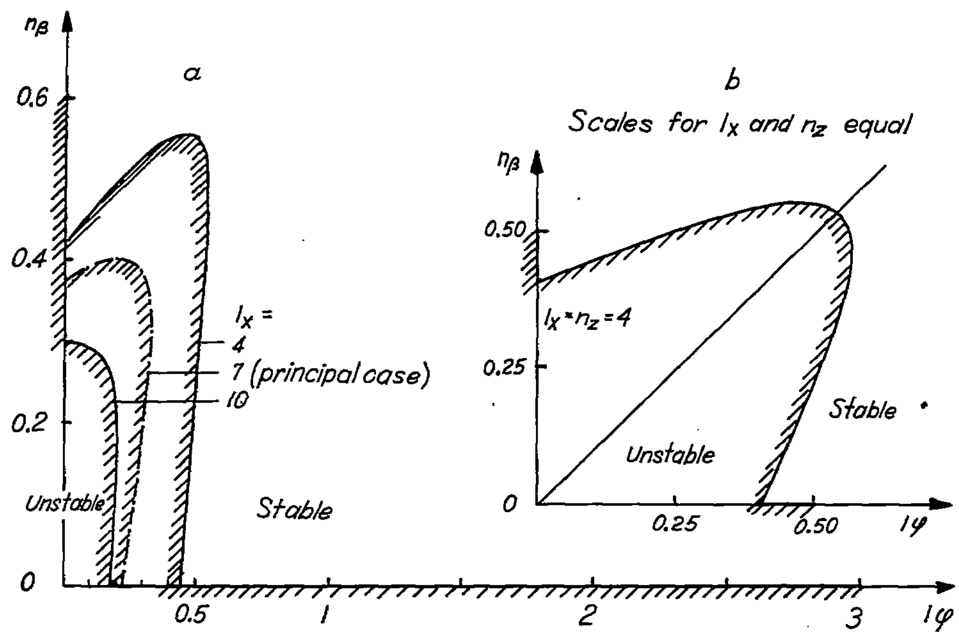

Figure 21(a) and (b).- Influence of the damping-in-roll l_x on the stability domain.

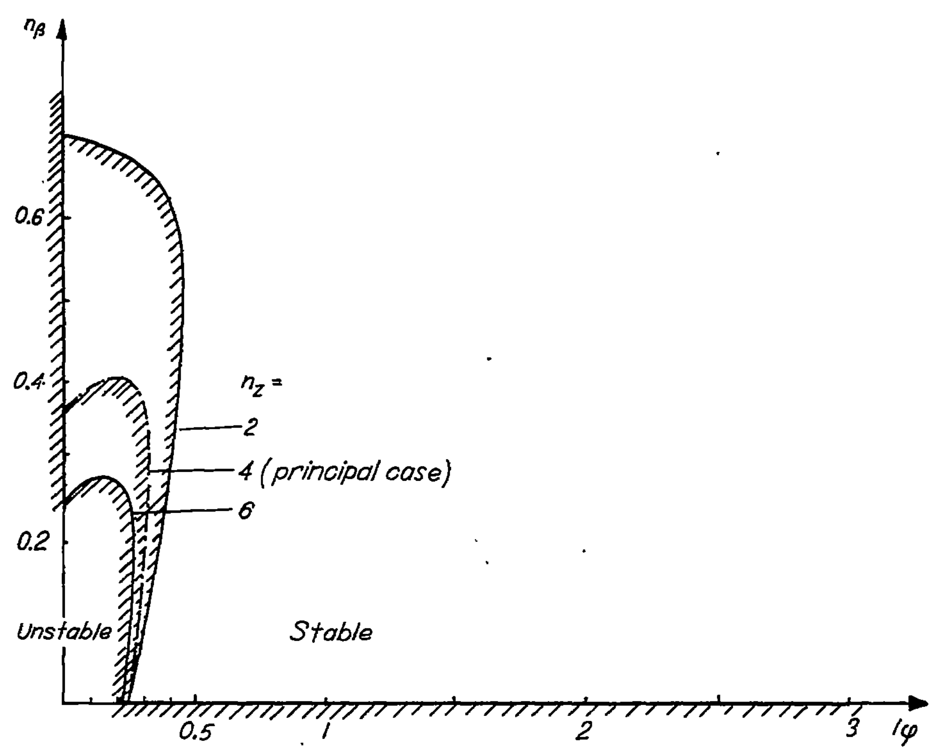

Figure 22.- Influence of the damping-in-yaw n_z on the stability domain.

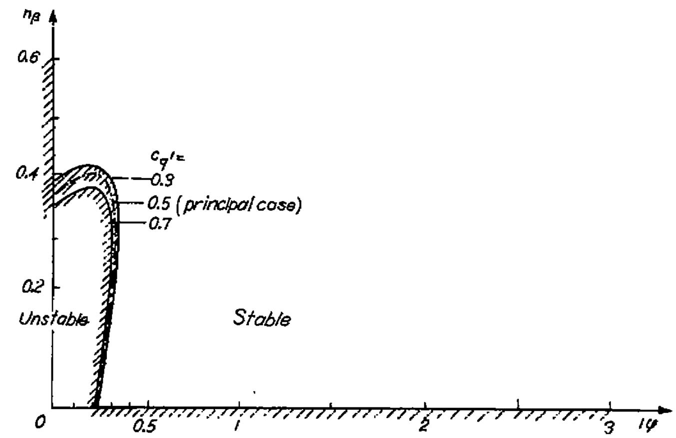

Figure 23.- Influence of the increase of the lateral force $c_q = \frac{\partial c_q}{\partial \beta}$ on the stability domain.

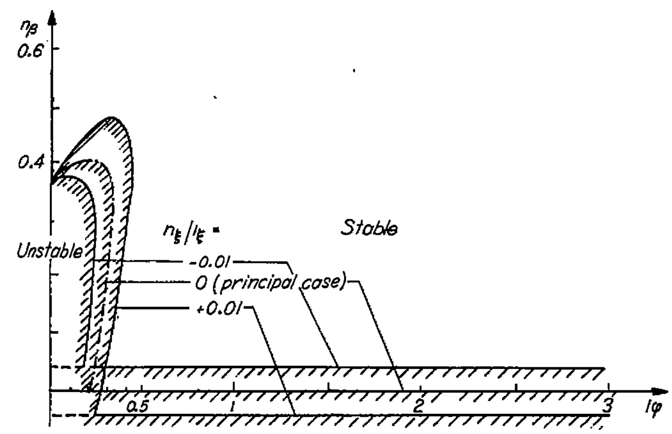

Figure 24.- Influence of the aileron yawing moment n_ξ on the stability domain.

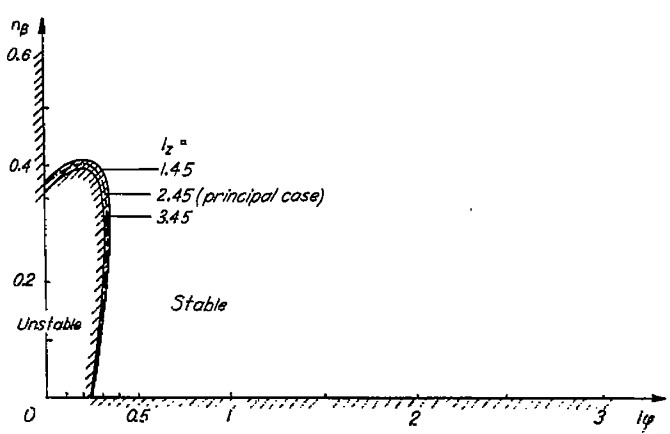

Figure 25.- Influence of the rolling moment due to yawing on the stability domain.

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